Mathematics for Decision Making: An Introduction

Lecture 5

Matthias Köppe

UC Davis, Mathematics

January 20, 2009

Case Study: Line Drawings on Pen Plotters

Optimizing the operation of a pen plotter

Pen plotters are used instead of printers for very large-scale line drawings, such as for drawings in architecture, or charts of logic circuits in electronics. (Nowadays pen plotters are gradually being replaced by large-format inkjet printers.)

- The plotter can move a pen horizontally
- At the same time it can roll the paper (either a large sheet or paper from a roll) up and down
- These movements can be done in pen-up (not drawing) or pen-down (drawing) mode

Problem: Given a drawing to be produced, minimize the total drawing time.

Key insights:

- How is the drawing time determined?
- There are two parts of the total drawing time – one part is independent of our decisions, one does depend on our decisions.
- Can we draw every drawing in pen-down mode only?
- What are useful variables for modeling?
- What constraints do we need?

Insights from Graph Theory

There are some line drawings that can be drawn entirely in pen-down mode, but most drawings require using pen-up mode as well – we need to move from one “dead end” to a new start.

We can gain understanding of this, again by using the mathematical abstraction of a **graph**.

- The vertices are the endpoints of (straight) lines of the drawing
- The edges represent the (straight) lines.

Theorem

In a graph, there is always an even number of odd nodes.

Theorem (Euler circuit)

A graph has an Euler circuit if and only if the degree of each vertex is even.

Theorem (Euler path)

A graph has an Euler path if and only if there are 0 or 2 vertices of odd degree.

Modeling the Drawing Time (Objective Function)

Consider a straight line (to be drawn in pen-down mode) or a movement (in pen-up mode) from point (x_1, y_1) to point (x_2, y_2). Let $(\Delta x, \Delta y) = (x_1 - x_2, y_1 - y_2)$.

Pen-down movement

We assume that the drawing time for the line depends on $|\Delta x|$ and $|\Delta y|$, so let's denote it by $f_{\text{down}}(\Delta x, \Delta y)$.
Depending on the "pen" technology (ink pens vs. ballpoint pens or cutting knives), a plausible requirement could be that all lines are drawn with the same pen velocity α, to ensure the same amount of ink bleeds into the paper. So the drawing time would be proportional to the Euclidean distance of the endpoints:

$$f_{\text{down}}(\Delta x, \Delta y) = \alpha\sqrt{(\Delta x)^2 + (\Delta y)^2}.$$

Pen-up movement

We denote the movement time by $f_{\text{up}}(\Delta x, \Delta y)$.
In pen-up mode, horizontal movement (moving the pen) and vertical movement (rolling the paper up or down) can be done independently, at the maximum speeds β, γ of the two (different!) technologies.
A plausible model of the movement time therefore is:

$$f_{\text{up}}(\Delta x, \Delta y) = \max\{\beta|\Delta x|, \gamma|\Delta y|\}$$