MidTerm 2 (Sample)

MAT 16A-4 Fall 2002

Name: ___________________________

PID: ___________________________

The actual exam will be shorter, but any of the problems similar to those below may show up.

1. ( ) Find an equation of the tangent line to
   a. the graph of \( f(x) = \sqrt{x} \) at point \( x = 9 \).
   b. the graph of \( g(x) = x^2 - x \) at point \( x = 2 \).
   c. the graph of \( h(x) = x^2 - 3 \) parallel to the line \( 2x + y = 2 \).

2. ( ) USE THE DEFINITION OF THE DERIVATIVE AT A POINT to find the derivatives of the following functions at indicated points:
   a. \( f(x) = (x + 3)^2, x = 2 \)
b. \( g(x) = 1/(1 - x), \quad x = 0 \)

3. () Differentiate the following functions
   a. \( f(x) = x^2 - 4x + 1 - \frac{2}{x} + \frac{1}{x^2} \)
   
   b. \( g(x) = \sqrt{x} - \sqrt{x^3} \)

4. ()
   a. Sketch the graph of \( \frac{4}{\sin(x/\pi)} \)
b. Find all solutions of $4\sin^2(2x) = 3$

c. Find solutions between $0$ and $2\pi$ of $\sin^2(2x) = 3\cos^2(2x)$

5. () Find first and second derivatives of the following functions:

a. $f(x) = \sin(x^2)$

b. $g(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$

c. $h(x) = \frac{x^2-1}{x^4+1}$
d. \( j(x) = \sin(\sin(x)) \)

e. \( k(x) = x \sin(x^3) \)

f. \( l(x) = \frac{\cos(\sqrt{x})}{x} \)

6. ( ) How many lines perpendicular to the bisectrix (the line which bisects an angle into two equal ones) of I and III quadrant and tangent to the graph of \( y = \frac{1}{2}x \) are there? Find their equations.

   Hint: Make a picture.
7. () Find equations of two tangent lines to the parabola $y = x^2$, which pass through the point with coordinates $(0, -15)$.

   Hint: Make a picture. Find an equation of the tangent line to the graph at a point on the graph with coordinates $(x_0, x_0^2)$. Find values for $x_0$ such that the line passes through the specified point $(0, -15)$

8. () The cost (in dollars) of the production of $q$ units of certain furniture is given by $C(q) = q^2 + 90q + 10,000$ and each unit sells for $1100$. Assuming an unlimited demand for the product find:

   a. Revenue and profit functions.

   b. Marginal cost, marginal revenue and marginal profit functions.
c. A production level (number of units) at which marginal profit is zero.

9. () An object is thrown upward with initial velocity 20m/s (meters per second) from certain height. It hits the ground after 5 seconds.

   Hint: The height in meters of an object at moment t (in seconds) projected upward with the initial velocity \( v_0 \) from the initial height \( h_0 \) is given by \( h(t) = -5t^2 + v_0 t + h_0 \).

   a. Find the initial height.

b. Find the maximum height of the object (when the object reaches its maximal height its velocity is equal to zero).
c. Find the velocity of the object when it hits the ground.

10. (a) Find $dy/dx$.
   
   a. $y^4 + xy^3 + x^2y^2 = x^3y + x^4$

   b. $x \sin(y) = y \cos(x)$

11. (a) Find an equation of the tangent line to the graph of $x^3 + y^3 = 28$ at point (1, 3).
12. (c) A 20 meter ladder is leaning against a wall. The base of the ladder is pulled away from the wall at a rate of 1 m/s (meter per second). How fast is the top of the ladder moving when the base 12 meters away from the wall.