1. Prove that
   \[ \lim_{x \to \infty} \frac{1}{x} \cos x = 0 \]

2. Compute limits.
   
   b) \[ \lim_{x \to 2} 7x + 3 = 17 \]

   a) \[ \lim_{x \to \infty} \frac{2x^2 + 5}{7x^2 + 4x + 3} \]
   
   b) \[ \lim_{x \to \infty} \frac{\sqrt{2x^3 + 4}}{x^2 + x + 1} \]
   
   c) \[ \lim_{x \to 1} \frac{x^3 - 1}{x - 1} \]
   
   d) \[ \lim_{x \to \infty} \frac{\sqrt{x^2 + 2x} - x}{x + 1} \]
   
   c) \[ \lim_{x \to 0} \frac{\sin(-x)}{\sin(5x)} \]
3) Prove that equation
\[ x^6 - 2x - 1 = 0 \] has at least one real root.

4) Determine at what points \( m = a, b, c, d \)

\( f(x) \)

I) There is a limit
\[ \lim_{x \to m} f(x) \]

II) \[ \lim_{x \to m^+} f(x) \]

III) \[ \lim_{x \to m^-} f(x) \]

5) Compute derivatives of
a) \[ \frac{\sin x + 1}{\cos x + 1} \]
b) \[ \frac{(\ln x)^3}{1 + x^2} \]
6) \( y(x) \) is a solution of
\[ \sin x \sin y + xy = \ln x \]
Find \( y' \) as a function of \( x \) and \( y \).

7) Find characteristic points
of function \( f(x) = \frac{\sqrt{x^2 - 1}}{x} \)
and plot the graph.

8) Using Newton method find
an first approximation to
the root of \( x^5 + x + 1 = 0 \).
Use \( x_0 = \frac{1}{2} \) as initial approximation.

9) The lengths of the two legs of a
right triangle depend on time. One leg,
whose length is \( x \), increases at a rate of \( 5 \)
feet per second, while the other of length \( y \), decreases at the rate of 10 feet per second. At what rate is the hypotenuse changing when \( x = 3 \) feet and \( y = 4 \) feet?

Is the hypotenuse increasing or decreasing then? Compute a rate of change of the area.

\( f(x) = x^2 e^{-x} \)

Find a) intercepts, b) critical points, c) local maxima minima d) inflection points, e) asymptotes. f) Graph the function.