Practice Final

MAT 21C Spring 2003. UCDavis, Instructor M. Movshev

Name: __________________________

PID: __________________________

The actual test will consist of ten problems. You will be given 2 hours to do the test. There will be no multiple choice problems. Calculators are allowed during the test. The actual final exam will be based on this practice final, both midterms and both practice midterms. You should expect to find problems similar to ones from previous examination you have had in this class.
1. ()

   a. Identify and sketch the surface given by

   \[ x^2 - 2y^2 + z^2 = 4 \]

   b. Draw level curves for the function

   \[ f(x, y) = (x + y)^2 - y^2 \]
2. ()
   
a. Use the definition of the limit of a real-valued function to prove that

\[
\lim_{{(x,y) \to (1,-1)}} 7x - 3y + 4 = 14
\]

b. Is

\[
g(x, y) = \begin{cases} 
\frac{x^2 + 2y^2}{x^2 + y^2} & (x, y) \neq (0, 0) \\
1 & (x, y) = (0, 0)
\end{cases}
\]  

(1)

continuous at the point \((0, 0)\)? State your position clearly and briefly justify it.
3. () Suppose that $R$ is the closed square having vertices $(-1, 1)$, $(1, 1)$, $(-1, 2)$, and $(1, 2)$ in the xy-plane. Find the global maximum and global minimum of

$$f(x, y) = (x + 3)^2 + (y + 2)^2$$

over the region $R$. Carefully justify your conclusion.
4. Let \( F(x, y) = xe^{x^2y} \), \( x = ab, y = \frac{9}{b} \). Carefully showing your work find

a. \( \frac{\partial F(x,y)}{\partial x \partial y} \)

b. \( \frac{\partial F(a,b)}{\partial a} \)
5. Compute the definite integral

\[ \int_{D} y^2 \, dx \, dy \]

where \( D \) is a region, enclosed by parabola \( x = y(y - 1) \) and \( y \) axes.
6. () For each of the following SET-UP the integral that would yield what is requested. DO NOT EVALUATE THE INTEGRALS; however, the given integral should be in a form for which all that remains to be carried out is the process of integration; all obvious simplifications should be carried out. Whenever one is needed, the density at a point \( P = (x, y, z) \) in the solid \( S \) is \( \sigma(P) = 3x^3 + y^2 \).

a. Let \( S \) be the solid that is bounded between the sphere \( x^2 + y^2 + z^2 \leq 1 \) and the half-space \( z \geq 1/2 \). SET-UP an iterated integral in \textbf{rectangular coordinates} that would yield the mass of \( S \).

b. SET-UP an iterated integral in \textbf{spherical coordinates} that would yield the volume of \( S \).
7. ()

a. Showing your work carefully, find the limit of \( \frac{1}{2 + n^{(-1)n}} \) if it exists.

b. 

\[
S_k = \sum_{n=1}^{k} \left( (n)\sin\left(\frac{1}{n}\right) - (n + 1)\sin\left(\frac{1}{n + 1}\right) \right)
\]

Find the partial sums \( S_2, S_3, S_4 \), then try to generalize the answer and find \( S_k \). After that compute \( \sum_{n=1}^{\infty} \left( (n)\sin\left(\frac{1}{n}\right) - (n + 1)\sin\left(\frac{1}{n + 1}\right) \right) \).
8. ( ) Determine whether each of the following converges (absolutely or conditionally) or diverges. Carefully justify your conclusions.

a. \[ \sum_{n=0}^{\infty} \left[ n \sin \left( \frac{1}{n} \right) \frac{(n+3)^3}{3^n} \right] \]

b. \[ \sum_{k=1}^{\infty} (-1)^k k^{-1} \ln^2(k) \]
c. \[ \sum_{k=1}^{\infty} \frac{1}{x^{1n(k)}} \]

d. \[ \sum_{k=1}^{\infty} \frac{k}{(2+(-1)^k)^k} \]
9. () Showing your work carefully, find the set of convergence of the series below and illustrate your conclusions on the number line:

\[ a. \sum_{n=2}^{\infty} \frac{(-1)^n}{2^{n} \ln(n)} (x - 1)^n \]

\[ b. \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{n!} (x + 10)^n \]
10. ()

a. Find the Taylor series expansion for \( f(x) = (2 - x)^{-2} \) about \( x = 3 \).

b. Find the Taylor series expansion for \( f(x) = (1 - x^2)^{-1} \) about \( x = 0 \).

c. Find the Taylor series expansion for \( f(x) = (1 - x^2)^{-1} \) about \( x = 2 \), by decomposing \( (1 - x^2)^{-1} \) into a sum \( a(1 - x)^{-1} + b(1 + x)^{-1} \) with some unknown \( a, b \).

d. Use a formula of the double angle to find \( n \)-th derivative of \( \cos^2(x) \) at \( x = 0 \)
11. ( ) Recall that \(\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}\) on interval \([-1, 1]\). So \(\frac{\pi}{2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}\)

a. Find an error of approximation of \(\arctan(x)\) by \(P_1(x, 0)\) on interval \([-1, 1]\).

b. Find an error of approximation of \(x^{1/2}\) by \(P_2(x, 1)\) on interval \([1, 2]\).
12. ()

a. Estimate the number \( N \) of terms one has to take in the partial sum \( S_N = \sum_{n=0}^{N} \frac{(-1)^n}{2n+1} \) to get an approximation to \( \frac{\pi}{4} \) exact in 3 digits.

b. It is known that \( \frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} \). Estimate the number \( N \) of terms one has to take in the partial sum \( S_N = \sum_{n=0}^{N} \frac{1}{n^2} \) to get an approximation to \( \frac{\pi^2}{6} \) exact in 3 digits.