Practice Midterm II

MAT 21C Spring 2003. UC Davis, Instructor M. Movshev

Name:

PID:

Solutions.

The actual test will consist of five problems. You will be given 50 minutes to do the test. There will be no multiple choice problems. Any notes or calculators are not allowed during the test.
2. (Section 15.3)

a. Find the center of a mass of the region bounded by $y = x^2$, $y = x$, $y = -x + 5$, $y = -2x + 10$, the density is $xy$

$$
\overline{x} = \frac{\int_{R} x^2y \, dA}{\int_{R} xy \, dA}, \quad \overline{y} = \frac{\int_{R} xy^2 \, dA}{\int_{R} xy \, dA}
$$

b. Find the kinetic energy of a metal plate in a form of a square, with vertexes at $(0,0),(0,1),(1,0),(1,1)$ and a mass density $\sigma(x,y) = xy$. The plate is rotated around $(1,0)$. The angular speed is 7 rad/sec.

$$
\text{note: kinetic energy} = \frac{1}{2} \left( \int_{R} \sigma(x,y) \, dA \right) \omega^2 \text{ joules}
$$

$$
\int_{R} r^2 \sigma(x,y) \, dA = \int_{0}^{1} \int_{0}^{1} \left( \sqrt{(x-1)^2 + y^2} \right)^2 \cdot xy \, dy \, dx = \int_{0}^{1} \int_{0}^{1} xy(x-1)^2 + xy^3 \, dy \, dx
$$

$$
= \int_{0}^{1} \left( \frac{y^2}{2} \cdot x(x-1)^2 \cdot \frac{x^3}{4} \right) \, dx = \int_{0}^{1} \left( \frac{x(x-1)^2}{2} + \frac{x}{4} \right) \, dx
$$

$$
= \left[ \frac{x^3}{2} - \frac{2x^2}{2} + \frac{3x}{4} \right]_{0}^{1} = \frac{x^4}{8} - \frac{2x^3}{6} + \frac{3x^2}{8}
$$

using formula, the kinetic energy is

$$
= \frac{1}{2} \left( \frac{1}{8} - \frac{2}{6} + \frac{3}{8} \right) \cdot 7^2
$$
4. (Section 15.5)

a. Draw the solid described \(1 \leq x \leq 4, 1 \leq y \leq 2, 0 \leq z \leq x\), setup a repeated integral for function \(f(x, y, z)\) over this domain

\[
\int_{0}^{1} \int_{1}^{2} \int_{z}^{4} f(x, y, z) \, dz \, dx \, dy
\]

b. Draw the solid described \(0 \leq y \leq 1, y^2 \leq x \leq y, 0 \leq z \leq x+y\), setup a repeated integral for function \(f(x, y, z)\) over this domain

\[
\int_{0}^{1} \int_{y^2}^{y} \int_{z}^{x+y} f(x, y, z) \, dz \, dx \, dy
\]

c. Integrate the function \(x^2 + y^2\) over the domain \(x \geq 0, y \leq 0, z \geq 0, \) and \(3x - 2y + 6z \leq 6\).

The plane \(z = 1 + \frac{x}{2} - \frac{y}{2}\) bounds solid from above

\[
\int_{0}^{1} \int_{\frac{3}{2}x - 3}^{\frac{3}{2}x - \frac{y}{2}} \int_{0}^{\frac{3}{2}x - y} (x^2 + y^2) \, dz \, dy \, dx
\]

etc. The integration is standard.
6. (Section 15.7)

a. Describe (by means of inequalities) in polar coordinates the regions bounded by two paraboloids \( z = x^2 + y^2 \) and \( z = 1 - x^2 - y^2 \).

\[ 0 \leq \Theta \leq 2\pi \\
0 \leq r \leq \frac{1}{\sqrt{2}} \\
1 - r^2 \leq z \leq r^2 \]

b. Find a volume of a sector of unit ball obtained by rotating a sector of a disk \( x^2 + z^2 \leq 1 \) and \( z \geq |x| \).

\[
V = \int_0^{2\pi} \int_0^{\pi/4} \int_0^1 p^2 \sin \phi \, dp \, d\phi \, d\Theta
\]

\[
= \left[ \int_0^{2\pi} \int_0^{\pi/4} \frac{p^3}{3} \sin \phi \, d\phi \right]_0^1 \, d\Theta
\]

\[
= \left[ \int_0^{2\pi} \frac{\sin \phi}{3} \, d\phi \right]_0^{\pi/4} \, d\Theta
\]

\[
= \left. \left[ -\cos \phi \right]_0^{\pi/4} \right) \, d\Theta
\]

\[
= \left. \left( -\frac{\sqrt{2}}{2} + 1 \right) \right) \, d\Theta
\]

\[
= \left. \left( -\frac{\sqrt{2} + 2}{2} \right) \right) \, d\Theta
\]

\[
= \left. \frac{2\pi}{6} \left( 2 - \frac{\sqrt{2} + 2}{2} \right) \right) \, d\Theta
\]

\[
= \frac{2\pi}{6} \left( 2 - \frac{\sqrt{2} + 2}{2} \right)
\]