1. (Section 15.2) Setup a formula of a repeated integral (without its evaluation) which computes 
\[ \iint f(x, y) \, dx \, dy \] 
where the domain of integration is bounded by curves

a. \( y = 4, y = 7, y = x + 1, y = 4x \)

We can describe our region \( 4 \leq y \leq 7 \) and \( \frac{9}{4} \leq x \leq y - 1 \)

\[ \int_{y=4}^{y=7} \int_{x=\frac{y-1}{4}}^{x=\frac{9}{4}} f(x,y) \, dx \, dy \]

b. \( x = 1, x = 4, y = 2x, y = x^2 + 3 \)

We can describe our region \( 1 \leq x \leq 4 \) and \( 2x \leq y \leq x^2 + 3 \)

\[ \int_{x=1}^{x=4} \int_{y=2x}^{y=x^2+3} f(x,y) \, dy \, dx \]

c. The region is bounded by ellipse \( x^2 + 5y^2 = 1 \)

\[ \frac{x^2}{\left( \frac{1}{\sqrt{5}} \right)^2} + \frac{y^2}{\left( \frac{1}{\sqrt{5}} \right)^2} = 1 \]

We can describe our region

\( -1 \leq x \leq 1 \) and \( -\sqrt{\frac{1-x^2}{5}} \leq y \leq \sqrt{\frac{1-x^2}{5}} \)

\[ \int_{x=-1}^{x=1} \int_{y=-\sqrt{\frac{1-x^2}{5}}}^{y=\sqrt{\frac{1-x^2}{5}}} f(x,y) \, dy \, dx \]
3. (Section 15.4)

a. Find an area of the domain $0 \leq \theta \leq 2\pi$ and $0 \leq r \leq \theta$, where $r$ is the distance from $(x, y)$ to the origin and $\theta$ is the angle of $(x, y)$ with $x$-axis.

\[ \int_0^{2\pi} \int_0^\theta r \, dr \, d\theta \]
\[ \int_0^{2\pi} \frac{r^2}{2} \, d\theta \bigg|_0^\theta = \int_0^{2\pi} \frac{\theta^2}{2} \, d\theta = \frac{\theta^3}{6} \bigg|_0^{2\pi} \]
\[ = \frac{8\pi^3}{6} = \frac{4\pi^3}{3} \]

b. Find the center of the mass of the region within the cardioid $r = 1 + \cos(\theta)$

\[ \overline{r} = 1 + \cos \theta \]

Assume $\sigma(P) = 1$.

By symmetry, $\overline{y} = 0$.

\[ X = \frac{\int_R x \sigma(P) \, dA}{\int_R \sigma(P) \, dA} \]
\[ = \frac{\int_0^{2\pi} \int_0^{1+\cos \theta} r \cos \theta \, r \, dr \, d\theta}{\int_0^{2\pi} \int_0^{1+\cos \theta} r \, dr \, d\theta} \]
\[ = \frac{\int_0^{2\pi} \int_0^{1+\cos \theta} \frac{r^3}{3} \cos \theta \, d\theta}{\int_0^{2\pi} \int_0^{1+\cos \theta} r \, dr \, d\theta} \]

\[ = \frac{\int_0^{2\pi} \frac{r^3}{3} \cos \theta \bigg|_0^{1+\cos \theta}}{\int_0^{2\pi} \frac{r^2}{2} \bigg|_0^{1+\cos \theta}} \]

Expand and apply trig identities.
5. (Section 15.6)

a. Draw the solid described in cylindrical coordinates $0 \leq \theta \leq \pi/2$, $0 \leq r \leq \cos(\theta)$, $1 \leq z \leq 2$. Setup a repeated integral for function $f(x, y, z)$ over this domain

$$
\int_1^2 \int_0^{\pi/2} \int_0^{\cos(\theta)} f(r \cos(\theta), r \sin(\theta), z) r \, dr \, d\theta \, dz
$$

b. Find a mass of a cylinder $0 \leq z \leq 1$, $x^2 + y^2 \leq 1$ with a mass density $\sigma(x, y, z) = z(x^2 + y^2)$.

The volume $V$ can be calculated as:

$$
V = \int_0^{2\pi} \int_0^1 \int_0^1 r^2 \, dr \, dz \, d\theta = \frac{2\pi}{3}
$$

The mass $M$ is given by:

$$
M = \int_0^{2\pi} \int_0^1 \int_0^1 z \sigma(r, \theta, z) r^2 \, dr \, dz \, d\theta
$$

After integrating, we get:

$$
M = \frac{2\pi}{3} \int_0^1 z \sigma(r, \theta, z) r^2 \, dr \, dz = \frac{2\pi}{3} \int_0^1 \frac{z^2}{4} \, dz = \frac{2\pi}{3} \int_0^1 \frac{z^2}{4} \, dz = \frac{\pi}{4}
$$
7. (Section 10.3)

a. Using the formula for the sum of geometric series compute

\[ \sum_{n=0}^{\infty} \frac{ar^n}{10^{n+3}} = \sum_{k=0}^{\infty} \left( \frac{15}{10^3} \right) \frac{1}{10^k} = \frac{\left( \frac{15}{10^3} \right)}{1 - \frac{1}{10}} \]

\[ = \frac{15}{10} \cdot \frac{10}{9} \]

\[ = \frac{15}{9 \cdot 10^2} \]

b. Using nth-term test for divergence determine convergence of the series

\[ \sum_{n=0}^{\infty} \frac{n^2 - 1}{n^2 + 1} \]

\[ \lim_{n \to \infty} \frac{n^2 - 1}{n^2 + 1} = \lim_{x \to \infty} \frac{x^2 - 1}{x^2 + 1} = 1 \]

\[ \Rightarrow \text{the series fails the } n^{th} \text{ term test, so it diverges} \]