Practice Midterm II

MAT 21C Spring 2003, UCDavis, Instructor M. Movshev

Name:________________________

PID:________________________

The actual test will consist of five problems. You will be given 50 minutes to do the test. There will be no multiple choice problems. Any notes or calculators are not allowed during the test.
1. (Section 15.2) Setup a formula of a repeated integral (without its evaluation) which computes \( \int \int f(x, y) \, dx \, dy \) where the domain of integration is bounded by curves

a. \( y = 4, y = 7, y = x + 1, y = 4x \)

b. \( x = 1, x = 4, y = 2x, y = x^2 + 3 \)

c. The region is bounded by ellipse \( x^2 + 5y^2 = 1 \)
2. (Section 15.3)

a. Find the center of a mass of the region bounded by \( y = x^2, \ y = x, \ y = -x + 5, \ y = -2x + 10 \), the density is \( xy \)

b. Find the kinetic energy of a metal plate in a form of a square, with vertexes at \((0,0),(0,1),(1,0),(1,1)\) and a mass density \( \sigma(x, y) = xy \). The plate is rotated around \((1,0)\). The angular speed is 7 rad/sec.
3. (Section 15.4)
   
   a. Find an area of the domain \(0 \leq \theta \leq 2\pi\) and \(0 \leq r \leq \theta\), where \(r\) is the distance from \((x, y)\) to the origin and \(\theta\) is the angle of \((x, y)\) with \(x\)-axis.

   b. Find the center of the mass of the region within the cardioid \(r = 1 + \cos(\theta)\)
4. (Section 15.5)

a. Draw the solid described $1 \leq x \leq 4, 1 \leq y \leq 2, 0 \leq z \leq x$, setup a repeated integral for function $f(x, y, z)$ over this domain

b. Draw the solid described $0 \leq y \leq 1, y^2 \leq x \leq y, 0 \leq z \leq x + y$, setup a repeated integral for function $f(x, y, z)$ over this domain

c. Integrate the function $x^2 + y^2$ over the domain $x \geq 0, y \leq 0, z \geq 0$, and $3x - 2y + 6z \leq 6$. 
5. (Section 15.6)

a. Draw the solid described in cylindrical coordinates \(0 \leq \theta \leq \pi/2, \ 0 \leq r \leq \cos(\theta), \ 1 \leq z \leq 2\). Setup a repeated integral for function \(f(x, y, z)\) over this domain.

b. Find a mass of a cylinder \(0 \leq z \leq 1, \ x^2+y^2 \leq 1\) with a mass density \(\sigma(x, y, z) = z(x^2+y^2)\).
6. (Section 15.7)

a. Describe (by means of inequalities) in polar coordinates the regions bounded by two paraboloids $z = x^2 + y^2$ and $z = 1 - x^2 - y^2$.

b. Find a volume of a sector of unit ball obtained by rotating a sector of a disk $x^2 + z^2 \leq 1$ and $z \geq |x|$. 
7. (Section 10.3)
   a. Using the formula for the sum of geometric series compute
      \[
      \sum_{n=0}^{\infty} \frac{15}{10^{n+3}}
      \]
   
   b. Using nth-term test for divergence determine convergence of the series
      \[
      \sum_{n=0}^{\infty} \frac{n^2 - 1}{n^2 + 1}
      \]