Math 108

Final Review Problems

- 1. Prove that $\{(x, y) \in \mathbf{N} \times \mathbf{R} : xy = 1\}$ is denumerable.
- 2. Prove that $(0,1] \cup (2,3] \cup (4,5)$ has cardinality **c**.
- 3. What is the cardinality of $\mathbf{R} (0, 1]$? Prove it.
- 4. Prove that $\mathbf{Q} \mathbf{Z}$ is denumerable.
- 5. Prove that if $\overline{\overline{A}} \leq \overline{\overline{B}}$ then $\overline{\overline{\mathscr{P}(A)}} \leq \overline{\overline{\mathscr{P}(B)}}$.
- 6. Order the following sets in terms of their cardinalities, from smallest to largest. Point out any ties.
 - $\mathbf{Q} \cup \{\pi\}$
 - $\mathbf{R} \{\pi\}$
 - $\mathscr{P}(\{0,1\})$
 - [0,2]
 - $(0,\infty)$
 - Z
 - $\mathbf{R} \mathbf{Z}$
 - $\mathscr{P}(\mathbf{R})$
- 7. Does there exist a one-one function $f: \mathscr{P}(\mathbf{N}) \to \mathbf{N}$? Prove your answer.