Math 16B Midterm 2 Fall 2011

NAME(print in CAPITAL letters, first name first): Key

NAME(sign): ____________________

ID#: ____________________

Instructions: There are eight problems. Some questions are easier than others so you are encouraged to read the entire exam before beginning your work. Make sure that you have all 8 problems.

1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

1. (20 points.) Find $\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx$.

$$\left[\begin{array}{l} u = 2x-1, \quad x = \tfrac{1}{2}(u+1) \\ \qquad\qquad dx = \tfrac{1}{2}\,du \\ x=1 \Rightarrow u=1 \\ x=5 \Rightarrow u=9 \end{array}\right.$$

$$= \int_1^9 \frac{\frac{1}{2}(u+1)}{\sqrt{u}}\,\frac{1}{2}\,du$$

$$= \frac{1}{4}\int_1^9 u^{1/2} + u^{-1/2}\,du$$

$$= \frac{1}{4}\left[\frac{2}{3}u^{3/2} + 2u^{1/2}\right]_1^9$$

$$= 16/3$$

$$\left[\begin{array}{l} u = \sqrt{2x-1}, \quad x = \tfrac{1}{2}(u^2+1) \\ \qquad\qquad dx = u\,du \\ x=1 \Rightarrow u=1 \\ x=5 \Rightarrow u=3 \end{array}\right.$$

$$= \int_1^3 \frac{\frac{1}{2}(u^2+1)}{u}\,u\,du$$

$$= \frac{1}{2}\int_1^3 u^2+1\,du$$

$$= \frac{1}{2}\left[\frac{1}{3}u^3+u\right]_1^3$$

$$= 16/3$$

2. (20 points.) Find $\int x\sqrt{x^2+1}\,dx$.

$$= \frac{1}{2}\int \underbrace{\sqrt{x^2+1}}_{\sqrt{u}}\;\underbrace{2x\,dx}_{du}$$

$$\left[\begin{array}{l} u = x^2+1 \\ du = 2x\,dx \end{array}\right.$$

$$= \frac{1}{2}\int \sqrt{u}\,du$$

$$= \frac{1}{3}u^{3/2} + C$$

$$= \frac{1}{3}(x^2+1)^{3/2} + C$$

3. (20 points.) Find $\int x^3 e^{x^2}\, dx$.

change variables $\left[\begin{array}{l} w = x^2 \\ dw = 2x\,dx \end{array} \right.$

$$= \frac{1}{2} \int \underbrace{x^2 e^{x^2}}_{we^w} \underbrace{2x\,dx}_{dw}$$

$$= \frac{1}{2} \int we^w\, dw$$

parts $\left[\begin{array}{ll} u = w & dv = e^w dw \\ du = dw & v = e^w \end{array} \right.$

$$= \frac{1}{2} \left[we^w - \int e^w\, dw \right]$$

$$= \frac{1}{2} \left[we^w - e^w \right] + C$$

4. (20 points.) Find $\int \frac{1}{x^2-1}\, dx$.

$$x^2-1 = (x+1)(x-1)$$

$$\frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$1 = A(x-1) + B(x+1)$$

If $x=1$,

$$1 = 2B \implies B = 1/2$$

If $x=-1$

$$1 = -2A \implies A = -1/2$$

$$\int \frac{dx}{x^2-1} = \int \frac{-1/2}{x+1} + \frac{1/2}{x-1}\, dx$$

$$= -1/2 \ln|x+1| + \frac{1}{2}\ln|x-1| + C$$

5. (20 points.) Find $\int x \sin x \, dx$.

parts $\left[\begin{array}{ll} u = x & dv = \sin x \, dx \\ du = dx & v = -\cos x \end{array}\right.$

$$= x(-\cos x) - \int -\cos x \, dx$$

$$= -x\cos x + \sin x + C$$

6. (20 points.) Find $\int_0^{\pi/4} \sin x \cos x \, dx$.

$$\left[\begin{array}{l} u = \sin x \\ du = \cos x \, dx \\ x = 0 \Rightarrow u = 0 \\ x = \pi/4 \Rightarrow u = \sqrt{2}/2 \end{array}\right.$$

$$= \int_0^{\frac{\sqrt{2}}{2}} u \, du$$

$$= \frac{1}{2} u^2 \Big]_0^{\sqrt{2}/2}$$

$$= \frac{1}{2} \left[\frac{1}{2} - 0\right] = 1/4$$

7. (20 points.) Find the area of the region bounded by the graphs of $y = 6/x$ and $y = 5 - x$.

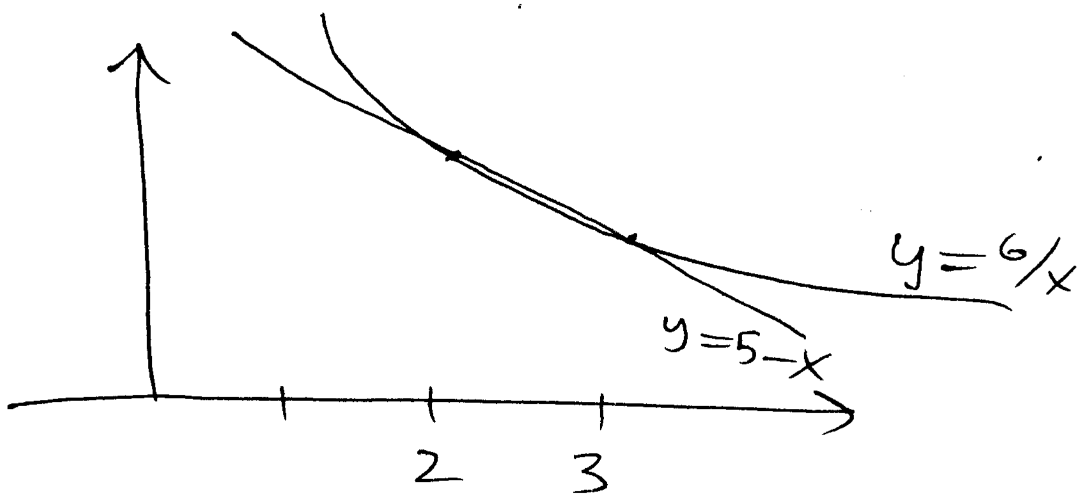
Find intersection points :

$$\frac{6}{x} = 5 - x$$

$$6 = 5x - x^2$$

$$x^2 + 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$



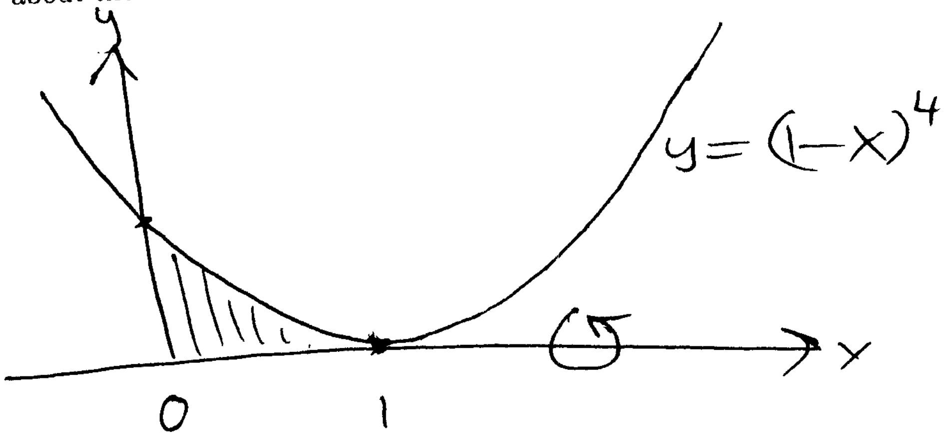
$$A = \int_2^3 5 - x - \frac{6}{x}\, dx$$

$$= 5x - \tfrac{1}{2}x^2 - 6\ln x \Big]_2^3$$

$$= \left(15 - \tfrac{9}{2} - 6\ln 3\right) - \left(10 - 2 - 6\ln 2\right)$$

$$= \tfrac{9}{2} + 6\ln 2 - 6\ln 3.$$

8. (20 points.) The region bounded by the graphs of $y = (1-x)^4$, $y = 0$ and $x = 0$ is revolved about the x-axis. Find the volume of the resulting solid.



$$V = \pi \int_0^1 \left[(1-x)^4 \right]^2 dx$$

$$= \pi \int_0^1 (1-x)^8 \, dx$$

$$\left[\begin{array}{ll} u = 1-x & x=0 \Rightarrow u=1 \\ du = -dx & x=1 \Rightarrow u=0 \end{array} \right.$$

$$= \pi \int_1^0 -u^8 \, du$$

$$= \pi \left[-\frac{1}{9} u^9 \right]_1^0$$

$$= \pi \left[0 - \left(-\frac{1}{9}\right) \right] = \pi/9$$