Math 16B **Problem Archive** **2010**

Instructions: This is a list of exam problems from previous years. WARNING: I make no promise that this year's questions will resemble these in any way. The following formulas will be given to you on the exams:

$$\begin{aligned}\sin A \sin B &= \frac{1}{2}(\cos(A-B) - \cos(A+B)) \\ \sin A \cos B &= \frac{1}{2}(\sin(A-B) + \sin(A+B)) \\ \cos A \cos B &= \frac{1}{2}(\cos(A-B) + \cos(A+B))\end{aligned}$$

$$\sin^2 A = \frac{1}{2}(1 - \cos(2A)), \quad \cos^2 A = \frac{1}{2}(1 + \cos(2A))$$

1. Solve the following equations for x.
   (a) $e^{-3x} = e$
   (b) $\frac{x^2}{2} = e^2$
   (c) $\frac{2}{2+10e^{0.1x}} = 5$
2. You have 10 pounds of a radioactive element whose half life is 60 months. How much time must elapse until only 1 pound remains? Give the final answer as a decimal using the approximation $\frac{\ln 5}{\ln 2} \approx 2.3$.
3. Evalulate the following integrals.
   (a) $\int \frac{x}{\sqrt{1+2x^2}} dx$
   (b) $\int \frac{e^x}{2} dx$
   (c) $\int \frac{1}{x(\ln x)^3} dx$
4. Find the derivatives of the following functions.
   (a) $f(x) = (\cos x)^{\ln(x)}$
   (b) $f(x) = \ln(x \ln x)$
5. Find the function f that satisfies $f''(x) = 2, f'(2) = 5$ and $f(2) = 10$.
6. Solve the following equations for x.
   (a) $e^x = 1$

(b) $x^{-2} = \frac{2}{e^2}$

(c) $e^{x+1} = 4$

7. You start with 81 pounds of a radioactive element. After 4 years, 1 pound remains. How many pounds remained after 3 years? Give a numerical answer.

8. Evalulate the following integrals.

   (a) $\int xe^{x^2} dx$

   (b) $\int \frac{1}{x \ln x} dx$

   (c) $\int \frac{e^x}{1+e^x} dx$

9. Find the derivatives of the following functions.

   (a) $f(x) = 10^{x^2}$

   (b) $f(x) = \ln(x\sqrt{4+x^2})$

10. Evaluate the following integrals.

    (a) $\int x^2 e^x dx$

    (b) $\int_0^e x^5 \ln x dx$

    (c) $\int \ln(3x) dx$

    (d) $\int \frac{4-3x}{(x-1)^2} dx$

11. Find the area between the graphs of $y = x^2 + 2x + 1$ and $y = 2x + 5$.

12. Find the volume when the region bounded by the graphs of $y = e^x$, $y = 0$, $x = 0$ and $x = 1$ is revolved about the x-axis.

13. (25 points.) Evaluate the following integrals.

    (a) $\int t \ln(t+1)\, dt$

    (b) $\int_2^5 \frac{x^2}{\sqrt{x-1}}\, dx$

14. (25 points.) Evaluate the following integrals.

    (a) $\int_0^4 \frac{x}{2x+1}\, dx$

    (b) $\int x \sec^2 x\, dx$

15. (25 points.) The region bounded by the graphs of $y = 1, y = 2$, $x = 1$ and $x = 2$ is revolved about the x-axis. Find the volume of the resulting solid.

16. (25 points.) Find the area under the graph of $y = \frac{-4}{x^2-x-6}$ between $x = -1$ and $x = 2$.

17. (20 points.) Evaluate the following integrals.

    (a) $\int \frac{e^x-1}{e^x+1}\,dx$

    (b) $\int \frac{\ln x}{x^2}\,dx$

18. (20 points.) Evaluate the following integrals.

    (a) $\int_{-1}^{1} x^3 e^{-x^2}\,dx$

    (b) $\int_0^1 (\frac{x}{x+1})^2\,dx$

19. (10 points.) Find a function f that satisfies

$$f''(x) = \frac{1}{x},\ x > 0;\quad f'(1) = 1;\quad f(1) = 1$$

20. (10 points.) Find the area between the graphs of $y = 8 - x^2$ and $y = x^2$.

21. (10 points.) Alice deposits \$50 into a bank account with an annual interest rate of 10%, compounded continuously. Bob deposits \$100 into an account with an annual interest rate of 5%, compounded coninuously. Give numerical answers to the following questions, using the approximation $\ln 2 \approx 0.7$.

    (a) How long does it take for Alice's money to double?

    (b) At what point do the two accounts have the same balance?

22. (10 points.) Use the trapezoidal rule with 4 subintervals to estimate $\int_0^1 e^{x^3}\,dx$. Do not simplify.