Instructions: This is a list of exam problems from previous years. WARNING: I make no promise that this year’s questions will resemble these in any way. The following formulas will be given to you on the exams:

\[
\sin A \sin B = \frac{1}{2} (\cos(A - B) - \cos(A + B))
\]

\[
\sin A \cos B = \frac{1}{2} (\sin(A - B) + \sin(A + B))
\]

\[
\cos A \cos B = \frac{1}{2} (\cos(A - B) + \cos(A + B))
\]

\[
\sin^2 A = \frac{1}{2} (1 - \cos(2A)), \quad \cos^2 A = \frac{1}{2} (1 + \cos(2A))
\]

1. Solve the following equations for \( x \).
   
   (a) \( e^{-3x} = e \)
   
   (b) \( \frac{x^2}{2} = e^2 \)
   
   (c) \( \frac{2}{2+10e^{0.5x}} = 5 \)

2. You have 10 pounds of a radioactive element whose half life is 60 months. How much time must elapse until only 1 pound remains? Give the final answer as a decimal using the approximation \( \ln 5 \approx \ln 2 \approx 2.3 \).

3. Evaluate the following integrals.
   
   (a) \( \int \frac{x}{\sqrt{1+2x^2}} \, dx \)
   
   (b) \( \int e^x \, dx \)
   
   (c) \( \int \frac{1}{x(\ln x)} \, dx \)

4. Find the derivatives of the following functions.
   
   (a) \( f(x) = (\cos x)^{\ln(x)} \)
   
   (b) \( f(x) = \ln(x \ln x) \)

5. Find the function \( f \) that satisfies \( f''(x) = 2, f'(2) = 5 \) and \( f(2) = 10 \).

6. Solve the following equations for \( x \).
   
   (a) \( e^x = 1 \)
(b) \( x^{-2} = \frac{2}{e^2} \)
(c) \( e^{x+1} = 4 \)

7. You start with 81 pounds of a radioactive element. After 4 years, 1 pound remains. How many pounds remained after 3 years? Give a numerical answer.

8. Evaluate the following integrals.
(a) \( \int xe^{x^2} \, dx \)
(b) \( \int \frac{1}{x \ln x} \, dx \)
(c) \( \int \frac{e^x}{1+e^x} \, dx \)

9. Find the derivatives of the following functions.
(a) \( f(x) = 10^x^2 \)
(b) \( f(x) = \ln(x\sqrt{4+x^2}) \)

10. Evaluate the following integrals.
(a) \( \int x^2 e^x \, dx \)
(b) \( \int_0^e x^5 \ln x \, dx \)
(c) \( \int \ln(3x) \, dx \)
(d) \( \int \frac{4-3x}{(x-1)^2} \, dx \)

11. Find the area between the graphs of \( y = x^2 + 2x + 1 \) and \( y = 2x + 5 \).

12. Find the volume when the region bounded by the graphs of \( y = e^x \), \( y = 0 \), \( x = 0 \) and \( x = 1 \) is revolved about the \( x \)-axis.

13. (25 points.) Evaluate the following integrals.
(a) \( \int t \ln(t+1) \, dt \)
(b) \( \int_2^5 \frac{x^2}{\sqrt{x-1}} \, dx \)

14. (25 points.) Evaluate the following integrals.
(a) \( \int_0^4 \frac{x}{2x+1} \, dx \)
(b) \( \int x \sec^2 x \, dx \)

15. (25 points.) The region bounded by the graphs of \( y = 1 \), \( y = 2 \), \( x = 1 \) and \( x = 2 \) is revolved about the \( x \)-axis. Find the volume of the resulting solid.

16. (25 points.) Find the area under the graph of \( y = \frac{-4}{x^2-1} \) between \( x = -1 \) and \( x = 2 \).
17. (20 points.) Evaluate the following integrals.
(a) \( \int \frac{e^x - 1}{x+1} \, dx \)
(b) \( \int \frac{\ln x}{x^2} \, dx \)

18. (20 points.) Evaluate the following integrals.
(a) \( \int_{-1}^{1} x^3 e^{-x^2} \, dx \)
(b) \( \int_{0}^{1} (\frac{x}{x+1})^2 \, dx \)

19. (10 points.) Find a function \( f \) that satisfies
\[ f''(x) = \frac{1}{x}, \; x > 0; \quad f'(1) = 1; \quad f(1) = 1 \]

20. (10 points.) Find the area between the graphs of \( y = 8 - x^2 \) and \( y = x^2 \).

21. (10 points.) Alice deposits $50 into a bank account with an annual interest rate of 10\%, compounded continuously. Bob deposits $100 into an account with an annual interest rate of 5\%, compounded continuously. Give numerical answers to the following questions, using the approximation \( \ln 2 \approx 0.7 \).

(a) How long does it take for Alice’s money to double?
(b) At what point do the two accounts have the same balance?

22. (10 points.) Use the trapezoidal rule with 4 subintervals to estimate \( \int_{0}^{1} e^{x^3} \, dx \). Do not simplify.