Math 16B **Midterm 2** **Fall 2010**

NAME(print in CAPITAL letters, first name first): Key

NAME(sign): _____

ID#: _____

Instructions: There are eight problems. Some questions are easier than others so you are encouraged to read the entire exam before beginning your work. Make sure that you have all 8 problems.

1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

$$\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$$
$$\sin A \cos B = \frac{1}{2}(\sin(A-B) + \sin(A+B))$$
$$\cos A \cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$$
$$\sin^2 A = \frac{1}{2}(1 - \cos(2A)), \quad \cos^2 A = \frac{1}{2}(1 + \cos(2A))$$

1. (20 points.) Find $\int_1^2 x\sqrt{x-1}\,dx$.

$$\begin{cases} u = x-1 \\ du = dx \end{cases}, \quad x = u+1$$

$x=1 \Rightarrow u=0$

$x=2 \Rightarrow u=1$

$$= \int_0^1 (u+1)\sqrt{u}\,du$$

$$= \int_0^1 u^{3/2} + u^{1/2}\,du$$

$$= \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\Big]_0^1$$

$$= \left(\frac{2}{5}+\frac{2}{3}\right) - 0 = \frac{16}{15}$$

2. (20 points.) Find $\int \frac{1}{\sqrt{x}+1}\,dx$.

$$\begin{cases} u = \sqrt{x}+1, & \sqrt{x} = u-1 \\ du = \frac{1}{2\sqrt{x}}\,dx \end{cases}$$

$$= \int \frac{1}{u}\,2(u-1)\,du$$

$$= 2\int 1 - \frac{1}{u}\,du$$

$$= 2(u - \ln|u|) + C$$

$$= 2\left(\sqrt{x}+1 - \ln\left|\sqrt{x}+1\right|\right) + C$$

3. (20 points.) Find $\int xe^x\,dx$.

parts $\begin{bmatrix} u = x & du = dx \\ dv = e^x dx & v = e^x \end{bmatrix}$

$= xe^x - \int e^x dx$

$= xe^x - e^x + C$

4. (20 points.) Find $\int \frac{\ln x}{x}\,dx$.

$\begin{bmatrix} u = \ln x \\ du = \frac{1}{x} dx \end{bmatrix}$

$= \int u\,du$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}(\ln x)^2 + C$

5. (20 points.) Find $\int \frac{1}{x(1-x)}\,dx$.

$$\frac{1}{x(1-x)} = \frac{A}{x} + \frac{B}{1-x}$$

$$1 = A(1-x) + Bx$$

$x=0$:

$$1 = A$$

$x=1$:

$$1 = B$$

$$\int \frac{1}{x(1-x)}\,dx = \int \frac{1}{x} + \frac{1}{1-x}\,dx$$

$$= \ln|x| - \ln|1-x| + C$$

6. (20 points.) Find $\int_0^{\pi/4} \sin x \cos x \, dx$.

$$\begin{aligned}
&u = \sin x \\
&du = \cos x \, dx \\
&x = 0 \Rightarrow u = 0 \\
&x = \frac{\pi}{4} \Rightarrow u = \frac{\sqrt{2}}{2}
\end{aligned}$$

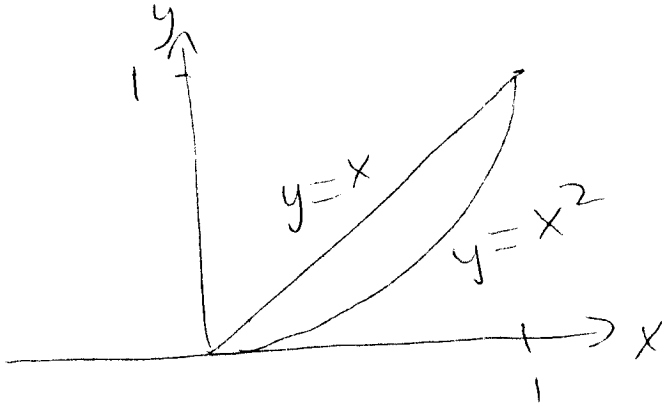
$$= \int_0^{\frac{\sqrt{2}}{2}} u \, du$$

$$= \frac{1}{2} u^2 \Big]_0^{\frac{\sqrt{2}}{2}}$$

$$= \frac{1}{2} \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2} \cdot \frac{2}{4}$$

$$= \frac{1}{4}$$

7. (20 points.) Find the area of the region bounded by the graphs of $y = x$ and $y = x^2$.



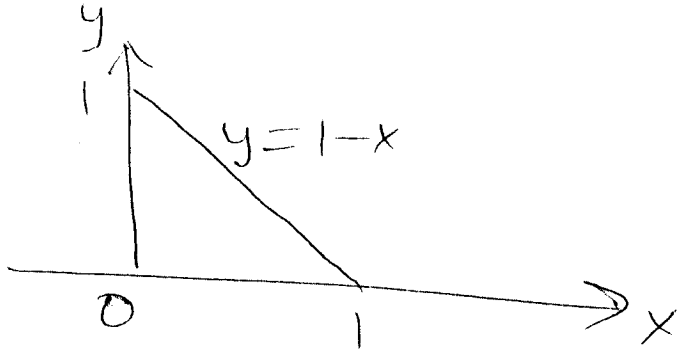
$$A = \int_0^1 x - x^2 \, dx$$

$$= \frac{1}{2}x^2 - \frac{1}{3}x^3 \Big]_0^1$$

$$= \left(\frac{1}{2} - \frac{1}{3}\right) - 0$$

$$= \frac{1}{6}$$

8. (20 points.) The region bounded by the graphs of $y = 1 - x$, $y = 0$ and $x = 0$ is revolved about the x-axis. Find the volume of the resulting solid.



Find intercept:

$$1 - x = 0$$

$$x = 1$$

$$V = \pi \int_0^1 (1-x)^2 \, dx$$

$$\begin{cases} u = 1 - x \\ du = -dx \\ x = 0 \Rightarrow u = 1 \\ x = 1 \Rightarrow u = 0 \end{cases}$$

$$= \pi \left(-\int_1^0 u^2 \, du \right)$$

$$= -\pi \left[\frac{1}{3} u^3 \right]_1^0 = -\pi \left[0 - \frac{1}{3} \right] = \frac{\pi}{3}$$