Math 21B-B - Homework Set 2

Section 5.3:

1. Express the following limits as definite integrals:

   (a) $\lim\limits_{\|P\|\to 0} \sum\limits_{k=1}^{n} ({c_k}^2 - 3c_k)\Delta x_k$, where P is a partition of $[-7,5]$

   (b) $\lim\limits_{\|P\|\to 0} \sum\limits_{k=1}^{n} \sqrt{4-{c_k}^2}\Delta x_k$, where P is a partition of $[0,1]$

   (c) $\lim\limits_{\|P\|\to 0} \sum\limits_{k=1}^{n} (\tan c_k)\Delta x_k$, where P is a partition of $[0,\pi/4]$

2. Suppose that f and g are integrable and that:

   $$\int_1^2 f(x)\,dx = -4, \qquad \int_1^5 f(x)\,dx = 6, \qquad \int_1^5 g(x) = 8.$$

   Use the rules in Table 5.4 to find:

   (a) $\int_2^2 g(x)\,dx$

   (b) $\int_5^1 g(x)\,dx$

   (c) $\int_1^2 3f(x)\,dx$

   (d) $\int_2^5 f(x)\,dx$

   (e) $\int_1^5 [f(x) - g(x)]\,dx$

   (f) $\int_1^5 [4f(x) - g(x)]\,dx$

3. Suppose that f and h are integrable and that:

   $$\int_1^9 f(x)\,dx = -1, \qquad \int_7^9 f(x)\,dx = 5, \qquad \int_7^9 h(x)\,dx = 4.$$

   Use the rules in Table 5.4 to find

   (a) $\int_1^9 -2f(x)\,dx$

   (b) $\int_7^9 [f(x) + h(x)]\,dx$

(c) $\displaystyle\int_7^9 [2f(x) - 3h(x)]\, dx$

(d) $\displaystyle\int_9^1 f(x)\, dx$

(e) $\displaystyle\int_1^7 f(x)\, dx$

(f) $\displaystyle\int_9^7 [h(x) - f(x)]\, dx$

4. Find $\displaystyle\int_{-2}^1 |x|\, dx$.

5. Find $\displaystyle\int_{-1}^1 (1 + \sqrt{1 - x^2})\, dx$.

6. Graph and find the average value of the following functions over the given interval

   (a) $f(x) = x^2 - 1$ on $[0, \sqrt{3}]$.

   (b) $h(x) = -|x|$ on

      i. $[-1, 0]$

      ii. $[0, 1]$

      iii. $[-1, 1]$

7. Use the method of example 4a to evaluate

   (a) $\displaystyle\int_a^b x^2\, dx,\ a < b$

   (b) $\displaystyle\int_{-1}^0 (x - x^2)\, dx$

8. Use the Max-Min Inequality to find upper and lower bounds for the value of

$$\int_0^1 \frac{1}{1 + x^2}\, dx.$$

9. Use the Max-Min Inequality to find upper and lower bounds for the the value of

$$\int_0^{0.5} \frac{1}{1 + x^2}\, dx \qquad \text{and} \qquad \int_{0.5}^1 \frac{1}{1 + x^2}\, dx.$$

10. Use the inequality $\sin x \leq x$, which holds for $x \geq 0$, to find an upper bound for the value of $\int_0^1 \sin x\, dx$.

Section 5.4:

1. Evaluate the following integrals:

   (a) $\int_{-2}^{0} (2x+5)\,dx$

   (b) $\int_{-3}^{4} \left(5 - \frac{x}{2}\right)\,dx$

   (c) $\int_{0}^{\pi} \sin x\,dx$

   (d) $\int_{0}^{\pi/3} 2\sec^2 x\,dx$

   (e) $\int_{1}^{-1} (r+1)^2\,dr$

   (f) $\int_{1}^{2} \left(\frac{1}{x} - e^{-x}\right)\,dx$

   (g) $\int_{0}^{1} \frac{4}{1+x^2}\,dx$

   (h) $\int_{2}^{5} \frac{x}{\sqrt{1+x^2}}\,dx$

   (i) $\int_{0}^{1} xe^{x^2}\,dx$

   (j) $\int_{1}^{2} \frac{\ln x}{x}\,dx$

2. Find the derivatives of the following functions (i) by evaluating the integral and differentiating the result, and (ii) by differentiating the integral directly:

   (a) $\frac{d}{dx} \int_{0}^{\sqrt{x}} \cos t\,dt$

   (b) $\frac{d}{d\theta} \int_{0}^{\tan\theta} \sec^2 y\,dy$

3. Find $\frac{dy}{dx}$ if $y = \int_{\sqrt{x}}^{0} \sin\left(t^2\right)\,dt$

4. In the exercises below, find the total area between the region and the x-axis:

   (a) $y = -x^2 - 2x$, where $-3 \le x \le 2$

   (b) $y = 3x^2 - 3$, where $-2 \le x \le 2$

(c) $y = x^{1/3} - x, \quad \text{where} \quad -1 \leq x \leq 8$

5. p. 335, problem 83.

6. Find $\lim_{x \to 0} \frac{1}{x^3} \int_0^x \frac{t^2}{t^4+1}\, dt$.

7. Suppose $f'(x) \geq 0$ for all values of x, and that $f(1) = 0$. Which of the following statements must be true of the function
$$g(x) = \int_0^x f(t)\, dt?$$
Give reasons for your answers.

   (a) g is a differentiable function of x.

   (b) g is a continuous function of x.

   (c) The graph of g has a horizontal tangent at $x = 1$.

   (d) g has a local maximum at $x = 1$.

   (e) g has a local minimum at $x = 1$.

   (f) The graph of g has an inflection point at $x = 1$.

   (g) The graph of dg/dx crosses the x-axis at $x = 1$.