Math 21B-B - Homework Set 4

Section 5.6:

1. Evaluate the following integrals.

   (a) $\int_0^{\frac{\pi}{4}} \tan x \sec^2 x \, dx$

   (b) $\int_{-\frac{\pi}{4}}^0 \tan x \sec^2 x \, dx$

2. Evaluate the following integrals.

   (a) $\int_0^{\sqrt{7}} t \left(t^2 + 1\right)^{1/3} dt$

   (b) $\int_{-\sqrt{7}}^0 t \left(t^2 + 1\right)^{1/3} dt$

3. Evaluate the following integrals.

   (a) $\int_0^1 \dfrac{x^3}{\sqrt{x^4 + 9}} \, dx$

   (b) $\int_{-1}^0 \dfrac{x^3}{\sqrt{x^4 + 9}} \, dx$

4. Evaluate the following integrals.

   (a) $\int_0^{\sqrt[3]{\pi^2}} \sqrt{\theta} \cos^2\left(\theta^{3/2}\right) d\theta$

   (b) $\int_{-1}^{-1/2} t^{-2} \sin^2\left(1 + \dfrac{1}{t}\right) dt$

5. Find $\int_2^4 \dfrac{dx}{x \ln x}$.

6. Find $\int_0^{\ln \sqrt{3}} \dfrac{e^x \, dx}{1 + e^{2x}}$.

7. Find the area between the graph of $y = x\sqrt{4 - x^2}$ and the x-axis.

8. Page 351, problem 52.

9. Page 351, problem 55.

10. Page 351, problem 58.

11. Find the area between the graphs of $y = 2x - x^2$ and $y = -3$.

12. Find the area between the graphs of $y = \sqrt{|x|}$ and $5y = x + 6$ (How many intersection points are there?).

13. Find the area between the graphs of $y = 3 - x^2$ and $y = -1$.

14. Suppose that $F(x)$ is an antiderivative of $f(x) = (\sin x)/x$, $x > 0$. Express

$$\int_1^3 \frac{\sin(2x)}{x}\,dx$$

in terms of F.

15. (a) Show that if f is odd on $[-a, a]$ then $\int_{-a}^a f(x)\,dx = 0$.

(b) Test the result in part (a) with $f(x) = \sin x$ and $a = \pi/2$.

16. A basic property of definite integrals is their invariance under translation, as expressed by the equation

$$\int_a^b f(x)\,dx = \int_{a-c}^{b-c} f(x + c)\,dx. \tag{1}$$

Use a substitution to verify Equation (1).