Math 21B-B - Homework Set 7

Section 7.1:

1. Evaluate the following integrals.

   (a)
   $$\int \frac{2y}{y^2 - 25}\, dy$$

   (b)
   $$\int \frac{\sec y \tan y}{2 + \sec y}\, dy$$

   (c)
   $$\int \frac{\sec x}{\sqrt{\ln(\sec x + \tan x)}}\, dx$$

   (d)
   $$\int \frac{e^{\sqrt{r}}}{\sqrt{r}}\, dr$$

   (e)
   $$\int \frac{e^{-1/x^2}}{x^3}\, dx$$

2. Solve the following initial value problems.

   (a) $\dfrac{dy}{dx} = 1 + \dfrac{1}{x}, \quad y(1) = 3$

   (b) $\dfrac{d^2y}{dx^2} = \sec^2 x, \quad y(0) = 0 \;\; y'(0) = 1$

3. **The linearization of** $\ln(1 + x)$ **at** $x = 0$**.** Instead of approximating $\ln x$ near $x = 1$ we approximate $ln(1+x)$ near $x = 0$. We get a simpler formula this way. Derive the linearization $\ln(1 + x) \approx x$ at $x = 0$.

4. **The linearization of** e^x **at** $x = 0$**.** Derive the linear approximation $e^x \approx 1 + x$ at $x = 0$.

Section 7.2:

1. **Atmospheric pressure.** The earth's atmospheric pressure p is often modeled by assuming that the rate dp/dh at which p changes with the altitue h above sea level is proportional to p. Suppose that the pressure at sea level is 1013 millibars (about 14.7 pounds per square inch) and that the pressure at an altitude of 20 km is 90 millibars.

(a) Solve the initial value problem

$$\frac{dp}{dh} = kp \quad (k\,\text{constant}) \qquad\qquad p = p_0 \text{ when } h = 0$$

(b) What is the atmospheric pressure at $h = 50$ km?

(c) At what altitude does the pressure equal 900 millibars?

2. **Voltage in a discharging capacitor.** Suppose that electricity is draining from a capacitor at a rate that is proportional to the voltage V across its terminals and that, if t is measured in seconds,

$$\frac{dV}{dt} = -\frac{1}{40}V.$$

Solve this equation for V, using V_0 to denote the value of V when $t = 0$. How long with it take the voltage to drip to 10% of its orginal value?

3. **Cholera bacteria.** Suppose that the bacteria in a colony can grow unchecked, by the law of exponential change. The colony starts with 1 bacterium and doubles every half-hour. How many bacteria will the colony contain at the end of 24 hours? (Under favorable laboratory conditions, the number of cholera bacteria can double every 30 min. In an infected person, many bacteria are destroyed, but this example explains why a person who feels well in the morning may be dangerously ill by evening.)

4. **Bank accounts with continuous compounding** Let $A(t)$ be the balance of a bank account at time t. Suppose that interest is compounded continuously with an annual interest rate r. This means that the differential equation

$$\frac{dA}{dt} = rA$$

is satisfied. This is the differential equation for exponential growth, and the solution is $A = A_0 e^{rt}$, where A_0 is the initial account balance. In this problem we explore some variations that lead to slightly different formulas.

(a) A bank account has an annual rate of return 0.05. You invest money in the account at a rate of \$1000 per year. If the initial balance is \$1000, when will the balance reach \$20,000? (Assume that investments are made continuously and that the return is compounded continuously.)

(b) A bank account has an annual rate of return $r = 0.10$. You invest at an annual rate of $1/A$ dollars per year when the account balance is A. Find the balance after 10 years if the initial balance is \$1. (Assume that investments are made continuously and that the return is compounded continuously.)

(c) A retirement account has an annual rate of return 10%. The broker charges fees at an annual rate of 5%. If the initial balance is \$1,0000, how much is collected in fees after 10 years? (Assume that the return and fees are computed continuously.)