21C: Final Exam

2014-07-31

Last name . . . .

First name . . . .

Email . . . .

1. (24 points.) For each series below, state whether it converges absolutely, converges conditionally, or diverges

a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{\ln n}{\ln(\ln n)}$

b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{2^n 3^n}{n^n}$

c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n^2+1}{2n^2+n-1}$

2. (24 points.) Consider the power series

$$\sum_{n=1}^{\infty} \frac{x^n}{e^{\sqrt{\ln n}}}.$$

a) Find the interval of convergence.

b) Find the radius of convergence.

c) Find all values of x such that the series is conditionally convergent.

3. (24 points.) For each problem below, if the limit exists, find it; otherwise state that the limit does not exist.

a) $$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$$

b) $$\lim_{(x,y)\to(1,1)} \frac{x^2-y^2}{x-y}$$

c) $$\lim_{(x,y)\to(1,-1)} \frac{x^3+y^3}{x+y}$$

4. (24 points.) Estimate $(1.01)^2(1.02)\cos\left(\frac{\pi}{2} + 0.02\right)$ to the nearest hundredth.

5. (24 points.) Find an equation for the tangent plane of the surface $x^2 + y^2 - z^2 = 18$ at the point $(3, 5, -4)$.

6. (24 points.) Find the directional derivative of $f(x, y, z) = \ln \sqrt{x^2 + y^2 + z^2}$ at the point $(3, 4, 12)$ in the direction of $\mathbf{v} = \langle 3, 6, -2 \rangle$.

7. (24 points.) Find the maximum and minimum value of $2x^2 - 8x + y^2$ subject to the constraint that $x^2 + y^2 \leq 9$.

8. (12 points.) Suppose that $0 < x < 1/4$. Which is larger,

a) $\sqrt{1+x} + \sqrt{1-x}$, or

b) $2 - x^2/4$?

(Hint: consider power series.)