NAME(print in CAPITAL letters, first name first): _______________________

NAME(sign): _______________________

ID#: _______________________

Instructions: There are six problems. Make sure that you have all 6 problems.

Points received:

1
2
3
4
5
6
TOTAL
1. (24 points.) For each series below, if the series converges then find its sum; otherwise, state that it diverges.

(a) \( \sum_{n=1}^{\infty} \left( \frac{1}{4} \right)^n \)

(b) \( \sum_{n=1}^{\infty} \left( \frac{5}{4} \right)^n \)

(c) \( \sum_{n=1}^{\infty} \frac{2^n + 3^n}{5^n} \)
2. (24 points.) For each series below, state whether it converges or diverges.

(a) \[ \sum_{n=2}^{\infty} \frac{1}{n \sqrt{\ln n}} \]

(b) \[ \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3} \]
3. (24 points.) For each series below, state whether it converges or diverges.

(a) \[ \sum_{n=1}^{\infty} \frac{n^2 + 3n + 1}{n^3 + 2n^2 + 3} \]

(b) \[ \sum_{n=1}^{\infty} \frac{n^2 - \cos n}{n^2 + \sin n} \]

(c) \[ \sum_{n=1}^{\infty} \frac{\ln n}{n} \]

(d) \[ \sum_{n=1}^{\infty} \frac{n^2 + (\ln n)^9}{n^4} \]
4. (24 points.) For each series below, state whether it converges absolutely, converges conditionally, or diverges

(a) \( \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n + \ln n}} \)

(b) \( \sum_{n=1}^{\infty} (-1)^n \frac{n + 1}{n^2 + 2n} \)

(c) \( \sum_{n=1}^{\infty} (-1)^n \sqrt{\frac{n}{n + 1}} \)

(d) \( \sum_{n=2}^{\infty} (-1)^n \frac{n^8}{n^{10} + e^{-n}} \)
5. (12 points.) Does the following series converge absolutely, converge conditionally, or diverge?

\[ \sum_{n=1}^{\infty} (-1)^n \left( \sqrt{n+1} - \sqrt{n} \right) \]
6. (24 points.) Consider the power series
\[
\sum_{n=1}^{\infty} \frac{n(x - 2)^n}{5^n}.
\]

(a) Find the interval of convergence.

(b) Find the radius of convergence.

(c) Find all values of \( x \) such that the series is conditionally convergent.