1. A fair die is rolled twice. Compute the joint p.m.f. of $X$ and $Y$, where $X$ is the first number rolled and $Y$ is the largest of the two numbers rolled.

2. Joint density of $(X, Y)$ is given by

$$f(x, y) = c(x^2 + \frac{xy}{2}), \quad 0 < x < 1, \quad 0 < y < 2.$$  

(a) Compute $c$.  
(b) Compute the density of $X$.  
(c) Compute $P(X > Y)$.  
(d) Compute the conditional probability $P(Y > 1/2 | X < 1/2)$.  
(e) Find $EY$.

3. Mr. Smith arrives at a location at a time uniformly distributed between 12:15 and 12:45, while Mrs. Smith independently arrives at the same location at a time uniformly distributed between 12 and 1 (all times p.m.).  
(a) Compute the probability that the first person to arrive waits no longer than 5 minutes.  
(b) Compute the probability that Mr. Smith arrives first.

4. Joint density of $(X, Y)$ is given by

$$f_1(x, y) = \begin{cases} 
  x \cdot e^{-(x+y)} & x, y > 0, \\
  0 & \text{otherwise}.
\end{cases}$$

(a) Are $X$ and $Y$ independent?  
(b) Repeat with joint density

$$f_2(x, y) = \begin{cases} 
  2 & 0 < x < y < 1, \\
  0 & \text{otherwise}.
\end{cases}$$

You should also do the five Problems in Section 7 of the book.
Solutions

1. For $i, j = 1, \ldots, 6$, $P(X = i, Y = j) = \begin{cases} \frac{1}{36} & \text{if } i < j, \\ \frac{1}{36} & \text{if } i = j, \\ 0 & \text{otherwise}. \end{cases}$

2. All detailed computations omitted, but you should do them! (a) Solve $\int_0^1 dx \int_0^2 f(x, y) dy = 1$ to get $c = \frac{6}{7}$. (b) $f_X(x) = \int_0^2 f(x, y) dy$. (c) $\int_0^1 dx \int_0^x f(x, y) dy$.
(d) $\frac{\int_0^{1/2} dx \int_0^{1/2} f(x, y) dy}{\int_0^{1/2} dx \int_0^2 f(x, y) dy}$.
(e) $\int_0^1 dx \int_0^y f(x, y) dy$.

3. Let the unit be 1 hour, $X$ uniform on $[1/4, 3/4]$, $Y$ uniform on $[0, 1]$ and independent. (a) $P(|X - Y| < 1/12) = 1/6$ (draw a picture). (b) $P(X \leq Y) = 1/2$.

4. (a) Yes, with marginal densities $f_X(x) = x \cdot e^{-x}$ and $f_Y(y) = e^{-y}$. (b) No, $(X, Y)$ is distributed uniformly, but not on a rectangle.