

# MAT 145 - Problem Set 2

## Due 5pm January 24

Collaboration is permitted; looking for solutions from external sources (books, the web, etc.) is prohibited.

1. Prove the following identity:

$$1 \cdot 2 + 2 \cdot 3 + \cdots + (n-1)n = \frac{(n-1)n(n+1)}{3}.$$

2. Draw  $n$  lines on the plane in such a way that no two are parallel and no three intersect in a common point. Prove that the plane is divided into  $\frac{n(n+1)}{2} + 1$  regions. (Hint: mathematical induction)
3. Show that  $\binom{2n}{n} \sim \frac{4^n}{\sqrt{\pi n}}$ .
4. In a class of all boys, 18 boys like to play chess, 23 like to play soccer, 21 like biking and 17 like hiking. The number of those who like to play both chess and soccer is 9. We also know that 7 boys like chess and biking, 6 boys like chess and hiking, 12 like soccer and biking, 9 boys like soccer and hiking, and finally 12 boys like biking and hiking. There are 4 boys who like chess, soccer, and biking, 3 who like chess, soccer, and hiking, 5 who like chess, biking, and hiking, and 7 who like soccer, biking, and hiking, Finally there are 3 boys who like all four activities. In addition we know that everybody likes at least one of these activities. How many boys are there in the class?
5. Consider numbers  $1, 2, \dots, 1000$ . Show that among any 501 of them there are two numbers such that one divides the other one. (Hint: pigeonhole principle)