

MAT 145 - Problem Set 3

due April 23

Collaboration is permitted; looking for solutions from external sources (books, the web, etc.) is prohibited.

1. Determine the number of permutations of $\{1, 2, \dots, 8\}$ in which no even integer is in its natural position.
2. Prove that in a group of $n > 1$ people there are two who have the same number of acquaintances in the group.
3. There are 100 people at a party. Each person has an even number of (possibly zero) acquaintances. Prove that there are three people at the party with the same number of acquaintances.
4. Prove the following inequalities:

$$\frac{n^k}{k^k} \leq \binom{n}{k} \leq \frac{n^k}{k!}.$$

5. In how many ways can you distribute n pennies to k children if each child is supposed to get at least 5?