Homework 4

Collaboration is permitted; looking for solutions from external sources (books, the web, etc.) is prohibited.

1. Prove that if we move straight down in Pascal's Triangle (visiting every other row), then the numbers we see are increasing.
2. Prove that
$$1 + \binom{n}{1}2 + \binom{n}{2}4 + \cdots + \binom{n}{n-1}2^{n-1} + \binom{n}{n}2^n = 3^n.$$
3. Prove that
$$\sum_{n_1+n_2+\cdots+n_k=n} \binom{n}{n_1, n_2, \ldots, n_k} = k^n$$
4. In how many ways can you cover a $2 \times n$ chessboard by dominoes? (Hint: Fibonacci recurrence)
5. Assume that the sequence $(a_0, a_1, a_2, \cdots)$ satisfies the recurrence $a_{n+1} = a_n + 2a_{n-1}$. We know that $a_0 = 4$ and $a_2 = 13$. What is a_5?
6. Prove that if n is a multiple of 4, then F_n (the nth Fibonacci number) is a multiple of 3.