MAT 145 - Problem Set
Due May 11th

Collaboration is permitted; looking for solutions from external sources (books, the web, etc.) is prohibited.

1. Show that a graph with at least two vertices has at least two vertices with the same degree.

2. A mouse intends to eat a $3 \times 3 \times 3$ cube of cheese. It begins at a corner and eats the whole of a $1 \times 1 \times 1$ cube before going to an adjacent one. Can the mouse end in the center?

3. For which values of n is it true that the complete graph K_n has an Eulerian walk?

4. Find a Hamiltonian cycle in the graph formed by the edges and vertices of an ordinary cube.

5. A *leaf* is a node of degree 1. Prove that if a tree has a node of degree d, then it has at least d leaves.