MAT 145 - Problem Set
Due May 18

Collaboration is permitted; looking for solutions from external sources (books, the web, etc.) is prohibited.

1. Prove that in every tree, any two paths with maximum length have a node in common.

2. Find all unlabeled trees on 2,3,4 and 5 nodes. How many labeled trees do you get from each? Use this to find the number of labeled trees on 2,3,4 and 5 nodes.

3. Does there exist an unlabeled tree with planar code

   (a) 111111000000;

   (b) 1010101010101010;

   (c) 1100011010?

4. Prove that if all edge-costs are different, then there is only one minimum spanning tree. (Optional hint: Suppose not. Then there are two minimum spanning trees. Let D be the set of edges in one tree but not in the other. Let e be the cheapest edge in D. Then e can be used to decrease the cost of one of the trees, which contradicts its optimality.)

5. Use Kruskal's algorithm to find the minimum spanning tree connecting the following big cities in the world: London, Mexico City, New York, Paris, Beijing and Tokyo. Distances in miles or kilometers can be found at `http://geobytes.com/CityDistanceTool/`.