Math 145 **Homework 8** **Spring 2018**

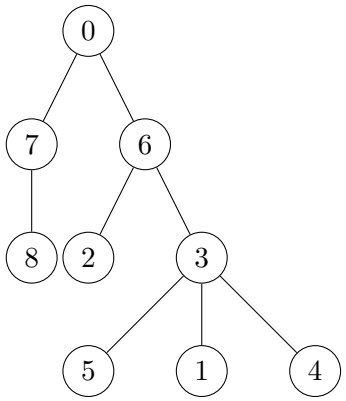
1. The *unit hypercube graph* H_n is defined as follows. The vertex set is the collection of binary strings of length n, and two vertices are joined by an edge when they differ in exactly one position. (For example in H_3 there is an edge from 010 to 110 because they differ in only the first position.)

   (a) How many edges are there in the graph H_n?

   (b) Is H_n bipartite?

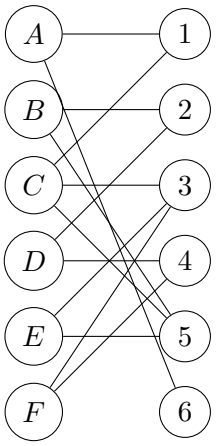
   (c) Does H_n have a perfect matching? If so, find one.

2. (a) Find the Prüfer code for the following tree.



   (b) Find the tree on vertices $\{0, 1, 2, 3, 4, 5, 6\}$ that has Prüfer code 04325 (that is, extended Prüfer code 043250).

3. Find a perfect matching in the following graph or prove that no perfect matching exists. (Optional hint: start with $\{A1, B2, C3, D4, E5\}$ and then run the augmenting path algorithm.)



4. Use Euler's formula to answer the following question. Into how many parts do the diagonals divide a convex n-gon? Assume no 3 diagonals go through the same point.

5. There are 3 houses and 3 wells. Can we build a path from every house to every well so that these paths do not cross? (The paths are not necessarily straight lines.)