Game theory homework 4

1. Find all pure Nash equilibria for the following general sum games.

   (a)
   $$\begin{bmatrix} (-3,-4) & (2,-1) & (0,6) & (1,1) \\ (2,0) & (2,2) & (3,0) & (1,-2) \\ (2,-3) & (-5,1) & (-1,-1) & (1,-3) \\ (-4,3) & (2,-5) & (1,2) & (-3,1) \end{bmatrix}.$$

   (b)
   $$\begin{bmatrix} (0,0) & (1,-1) & (1,1) & (-1,0) \\ (-1,1) & (0,1) & (1,0) & (0,0) \\ (1,0) & (-1,-1) & (0,1) & (-1,1) \\ (1,-1) & (-1,0) & (1,-1) & (0,0) \\ (1,1) & (0,0) & (-1,-1) & (0,0) \end{bmatrix}.$$

2. Two smart students form a study group in a Math class where homeworks are handed in jointly by each study group. In the last homework of the quarter, each of the two students can choose to work ("W") or defect ("D"). If at least one of them solves the homework that week (chooses "W"), then they will both receive 10 points. But solving the homework incurs an effort worth -7 points for a student doing it alone and an effort worth -2 points for each student if both students work together. Assume that the students do not communicate prior to deciding whether they will work or defect. Write this situation as a matrix game and determine all Nash equilibria.

3. **Two cheetahs and three antelopes:** Two cheetahs each chase one of three antelopes. If they catch the same one, they have to share. The antelopes are Large, Small and Tiny, and their values to the cheetahs are l, s, and t. Write the 3×3 matrix for this game. Assume that $t < s < l < 2s$, and that
   $$\frac{l}{2}\left(\frac{2l-s}{l+s}\right) + l\left(\frac{2s-l}{l+s}\right) < t. \tag{1}$$
   Find the pure equilibria, and the symmetric mixed equilibria. (Hint: the left-hand side of the inequality is the mean payoff to each player for the symmetric Nash equilibrium of the game in Example 3.2.2 of Karlin and Peres.)

4. Three firms (players I, II, and III) put three items on the market and advertise them either on morning or evening TV. A firm advertises exactly once per day. If more than one firm advertises at the same time, their profits are zero. If exactly one firm advertises in the morning, its profit is $\$200$K. If exactly one firm advertises in the evening, its profit is $\$300$K. Firms must take their advertising decisions simultaneously. Find a symmetric mixed Nash equilibrium.