

Math 21B-B - Homework Set 3

Section 4.8:

4. a. $f(x) = 2x^{-3}$
 $F(x) = -x^{-2}$
- b. $f(x) = \frac{x^{-3}}{2} + x^2$
 $F(x) = -\frac{x^{-2}}{4} + \frac{1}{3}x^3$
- c. $f(x) = -x^{-3} + x - 1$
 $F(x) = \frac{1}{2}x^{-2} + \frac{1}{2}x^2 - x$
10. a. $f(x) = \frac{1}{2}x^{-1/2}$
 $F(x) = x^{1/2}$
- b. $f(x) = -\frac{1}{2}x^{-3/2}$
 $F(x) = x^{-1/2}$
- c. $f(x) = -\frac{3}{2}x^{-5/2}$
 $F(x) = x^{-3/2}$
13. a. $f(x) = -\pi \sin(\pi x)$
 $F(x) = \cos(\pi x)$
- b. $f(x) = 3 \sin(x)$
 $F(x) = -3 \cos(x)$
- c. $f(x) = \sin(\pi x) - 3 \sin(3x)$
 $F(x) = -\frac{1}{\pi} \cos(\pi x) + \cos(3x)$
15. a. $f(x) = \sec^2(x)$
 $F(x) = \tan(x)$
- b. $f(x) = \frac{2}{3} \sec^2\left(\frac{x}{3}\right)$
 $F(x) = 2 \tan\left(\frac{x}{3}\right)$
- c. $f(x) = -\sec^2\left(\frac{3x}{2}\right)$
 $F(x) = -\frac{2}{3} \tan\left(\frac{3x}{2}\right)$

18. a. $f(x) = \sec(x) \tan(x)$

$$F(x) = \sec(x)$$

b. $f(x) = 4 \sec(3x) \tan(3x)$

$$F(x) = \frac{4}{3} \sec(3x)$$

c. $f(x) = \sec\left(\frac{\pi x}{2}\right) \tan\left(\frac{\pi x}{2}\right)$

$$F(x) = \frac{2}{\pi} \sec\left(\frac{\pi x}{2}\right)$$

19. a. $f(x) = e^{3x}$

$$F(x) = \frac{1}{3} e^{3x}$$

b. $f(x) = e^{-x}$

$$F(x) = -e^{-x}$$

c. $f(x) = e^{x/2}$

$$F(x) = 2e^{x/2}$$

27. $\int \left(3t^2 + \frac{t}{2}\right) dt = t^3 + \frac{t^2}{4} + C$

$$\frac{d}{dt} \left(t^3 + \frac{t^2}{4} + C\right) = 3t^2 + 2 \cdot \frac{t}{4} + 0 = 3t^2 + \frac{t}{2}$$

36. $\int \left(\frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}}\right) dx = \frac{x^{3/2}}{3} + 4\sqrt{x} + C$

$$\frac{d}{dx} \left(\frac{x^{3/2}}{3} + 4\sqrt{x} + C\right) = \frac{3}{2} \cdot \frac{x^{1/2}}{3} + \frac{1}{2} \cdot 4x^{-1/2} + 0 = \frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}}$$

45. $\int \left(7 \sin\left(\frac{\theta}{3}\right)\right) d\theta = -21 \cos\left(\frac{\theta}{3}\right) + C$

$$\frac{d}{d\theta} \left(-21 \cos\left(\frac{\theta}{3}\right) + C\right) = \frac{1}{3} \cdot 21 \sin\left(\frac{\theta}{3}\right) + 0 = 7 \sin\left(\frac{\theta}{3}\right)$$

52. $\int (2e^x - 3e^{-2x}) dx = 2e^x + \frac{3}{2}e^{-2x} + C$

$$\frac{d}{dx} (2e^x + \frac{3}{2}e^{-2x} + C) = 2e^x + (-2) \cdot \frac{3}{2}e^{-2x} + 0 = 2e^x - 3e^{-2x}$$

65. $\int (1 + \tan^2 \theta) d\theta = \int \sec^2 \theta d\theta = \tan \theta + C$

$$\frac{d}{d\theta} (\tan \theta + C) = \sec^2 \theta + 0 = 1 + \tan^2 \theta$$

$$66. \int (2 + \tan^2 \theta) d\theta = \int (1 + \sec^2 \theta) d\theta = \theta + \tan \theta + C$$

$$\frac{d}{d\theta} (\theta + \tan \theta + C) = 1 + \sec^2 \theta + 0 = 1 + 1 + \tan^2 \theta = 2 + \tan^2 \theta$$

72.

$$\begin{aligned} \frac{d}{dx} \left(-\frac{(3x+5)^{-1}}{3} + C \right) &= \frac{d}{dx} \left(-\frac{1}{3}(3x+5)^{-1} + C \right) \\ &= -\frac{1}{3} \cdot -(3x+5)^{-2}(3) \\ &= (3x+5)^{-2} \end{aligned}$$

75.

$$\begin{aligned} \frac{d}{dx} \left(-\frac{1}{x+1} + C \right) &= \frac{d}{dx} [-(x+1)^{-1} + C] \\ &= (x+1)^{-2} \\ &= \frac{1}{(x+1)^2} \end{aligned}$$

83. a. Wrong.

b. Wrong.

c. Right.

84. a. Wrong.

b. Right.

c. Right.

125. Suppose that $f(x) = \frac{d}{dx}(1 - \sqrt{x})$ and $g(x) = \frac{d}{dx}(x + 2)$.

$$a. \int f(x) dx = 1 - \sqrt{x} + C = -\sqrt{x} + C$$

$$b. \int g(x) dx = x + 2 + C = x + C$$

$$c. \int [-f(x)] dx = -\int f(x) dx = -1 + \sqrt{x} + C = \sqrt{x} + C$$

$$d. \int [-g(x)] dx = -\int g(x) dx = -x - 2 + C = -x + C$$

$$\text{e. } \int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx = 1 - \sqrt{x} + x + 2 + C = -\sqrt{x} + x + C$$

$$\text{f. } \int [f(x) - g(x)] dx = \int f(x) dx - \int g(x) dx = 1 - \sqrt{x} - x - 2 + C = -\sqrt{x} - x + C$$

Section 5.5:

2. $\int x \sin(2x^2) dx.$

Let $u = 2x^2$. Then $du = 4x dx \Rightarrow dx = \frac{1}{4x} du.$

$$\begin{aligned} \int x \sin(2x^2) dx &= \int \frac{1}{4} \sin(u) du \\ &= -\frac{1}{4} \cos(u) + C \\ &= -\frac{1}{4} \cos(2x^2) + C. \end{aligned}$$

5. $\int 28(7x - 2)^{-5} dx.$

Let $u = 7x - 2$. The $du = 7 dx \Rightarrow dx = \frac{1}{7} du.$

$$\begin{aligned} \int 28(7x - 2)^{-5} dx &= \int 4u^{-5} du \\ &= -u^{-4} + C \\ &= -(7x - 2)^{-4} + C \end{aligned}$$

7. $\int \frac{9r^2 dr}{\sqrt{1 - r^3}}.$

Let $u = 1 - r^3$. Then $du = -3r^2 dr \Rightarrow dr = -\frac{1}{3r^2} du.$

$$\begin{aligned} \int \frac{9r^2 dr}{\sqrt{1 - r^3}} &= \int -\frac{3 du}{\sqrt{u}} \\ &= \int -3u^{-1/2} du \\ &= -6u^{1/2} + C \\ &= -6\sqrt{1 - r^3} + C \end{aligned}$$

$$10. \int \frac{1}{x^2} \cos^2 \left(\frac{1}{x} \right) dx.$$

$$\text{Let } u = \frac{1}{x}. \text{ Then } du = -\frac{1}{x^2} dx \Rightarrow dx = -x^2 du.$$

$$\begin{aligned} \int \frac{1}{x^2} \cos^2 \left(\frac{1}{x} \right) dx &= \int -\cos^2 u \, du \\ &= \int -\left[\frac{1}{2} + \frac{\cos(2u)}{2} \right] du \quad (\text{Half-Angle Formula}) \\ &= -\frac{u}{2} - \frac{\sin(2u)}{4} + C \\ &= -\frac{1}{2x} - \frac{1}{4} \sin \left(\frac{2}{x} \right) + C. \end{aligned}$$

$$13. \int \sqrt{3-2s} \, ds.$$

$$\text{Let } u = 3-2s. \text{ Then } du = -2 \, ds \Rightarrow ds = -\frac{1}{2} du$$

$$\begin{aligned} \int \sqrt{3-2s} \, ds &= \int -\frac{1}{2} \sqrt{u} \, du \\ &= -\frac{1}{3} u^{3/2} + C \\ &= -\frac{1}{3} (3-2s)^{3/2} + C. \end{aligned}$$

$$18. \int \frac{4y \, dy}{\sqrt{2y^2+1}}.$$

$$\text{Let } u = 2y^2+1. \text{ Then } du = 4y \, dy \Rightarrow dy = \frac{1}{4y} du.$$

$$\begin{aligned} \int \frac{4y \, dy}{\sqrt{2y^2+1}} &= \int \frac{1}{\sqrt{u}} \, du \\ &= 2\sqrt{u} + C \\ &= 2\sqrt{2y^2+1} + C \end{aligned}$$

$$23. \int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx.$$

$$\text{Let } u = \cos x. \text{ Then } du = -\sin x \, dx \Rightarrow dx = -\frac{1}{\sin x} du.$$

$$\begin{aligned}
\int \frac{\sin x}{\cos x} dx &= \int -\frac{1}{u} du \\
&= -\ln|u| + C \\
&= -\ln|\cos x| + C \\
&= \ln|\sec x| + C. \quad (\text{Properties of ln})
\end{aligned}$$

28. $\int x^{1/3} \sin(x^{4/3} - 8) dx.$

Let $u = x^{4/3} - 8$. Then $du = \frac{4}{3}x^{1/3} dx \Rightarrow dx = \frac{3}{4}x^{-1/3} du.$

$$\begin{aligned}
\int x^{1/3} \sin(x^{4/3} - 8) dx &= \int \frac{3}{4} \sin u du \\
&= -\frac{3}{4} \cos u + C \\
&= -\frac{3}{4} \cos(x^{4/3} - 8) + C.
\end{aligned}$$

37. $\int \sqrt{\frac{x-1}{x^5}} dx = \int \frac{1}{x^2} \sqrt{\frac{x-1}{x}} dx$

Let $u = \frac{x-1}{x}$. Then $du = \frac{1}{x^2} dx \Rightarrow dx = x^2 du$

$$\begin{aligned}
\int \frac{1}{x^2} \sqrt{\frac{x-1}{x}} dx &= \int \sqrt{u} du \\
&= \frac{2}{3} u^{3/2} + C \\
&= \frac{2}{3} \left(\frac{x-1}{x} \right)^{3/2} + C
\end{aligned}$$

39. $\int (\cos x) e^{\sin x} dx$

Let $u = \sin x$. Then $du = \cos x dx \Rightarrow dx = \frac{du}{\cos x}$

$$\begin{aligned}
\int (\cos x) e^{\sin x} dx &= \int e^u du \\
&= e^u + C \\
&= e^{\sin x} + C
\end{aligned}$$

$$42. \int \frac{1}{x^2} e^{1/x} \sec(1 + e^{1/x}) \tan(1 + e^{1/x}) dx$$

$$\text{Let } u = 1 + e^{1/x}. \text{ Then } du = e^{1/x} \cdot -\frac{1}{x^2} dx \Rightarrow dx = -x^2 e^{-1/x} du$$

$$\begin{aligned} \int \frac{1}{x^2} e^{1/x} \sec(1 + e^{1/x}) \tan(1 + e^{1/x}) dx &= \int -\sec u \tan u du \\ &= -\sec u + C \\ &= -\sec(1 + e^{1/x}) + C \end{aligned}$$

$$43. \int \frac{dx}{x \ln x}$$

$$\text{Let } u = \ln x. \text{ Then } du = \frac{1}{x} dx \Rightarrow dx = x du$$

$$\begin{aligned} \int \frac{dx}{x \ln x} &= \int \frac{du}{u} \\ &= \ln u + C \\ &= \ln |\ln x| + C \end{aligned}$$

$$48. \int \frac{1}{\sqrt{e^{2\theta} - 1}} d\theta = \int \frac{e^\theta}{e^\theta \sqrt{e^{2\theta} - 1}} d\theta$$

$$\text{Let } u = e^\theta. \text{ Then } du = e^\theta d\theta \Rightarrow d\theta = e^{-\theta} du$$

$$\begin{aligned} \int \frac{e^\theta}{e^\theta \sqrt{e^{2\theta} - 1}} d\theta &= \int \frac{du}{u \sqrt{u^2 - 1}} \\ &= \sec^{-1} u + C \\ &= \sec^{-1}(e^\theta) + C \end{aligned}$$

$$55. \int \frac{18 \tan^2 x \sec^2 x}{(2 + \tan^3 x)^2} dx$$

$$\text{a. } u = \tan x, \quad du = \sec^2 x dx$$

$$\int \frac{18 \tan^2 x \sec^2 x}{(2 + \tan^3 x)^2} dx = \int \frac{18u^2}{(2 + u^3)^2} du$$

$$v = u^3, \quad dv = 3u^2 du$$

$$\int \frac{18u^2}{(2+u^3)^2} du = \int \frac{6}{(2+v)^2} dv$$

$$w = 2 + v, \quad dw = dv$$

$$\begin{aligned} \int \frac{6}{(2+v)^2} dv &= \int \frac{6}{w^2} dw \\ &= -\frac{6}{w} + C \\ &= -\frac{6}{2+v} + C \\ &= -\frac{6}{2+u^3} + C \\ &= -\frac{6}{2+\tan^3 x} + C \end{aligned}$$

b. $u = \tan^3 x, \quad du = 3 \tan^2 x \cdot \sec^2 x \, dx$

$$\int \frac{18 \tan^2 x \sec^2 x}{(2 + \tan^3 x)^2} dx = \int \frac{6}{(2+u)^2} du$$

$$v = 2 + u, \quad dv = du$$

$$\begin{aligned} \int \frac{6}{(2+u)^2} du &= \int \frac{6}{v^2} dv \\ &= -\frac{6}{v} + C \\ &= -\frac{6}{2+u} + C \\ &= -\frac{6}{2+\tan^3 x} + C \end{aligned}$$

c. $u = 2 + \tan^3 x, \quad du = 3 \tan^2 x \cdot \sec^2 x \, dx$

$$\begin{aligned} \int \frac{18 \tan^2 x \sec^2 x}{(2 + \tan^3 x)^2} dx &= \int \frac{6}{u^2} du \\ &= -\frac{6}{u} + C \\ &= -\frac{6}{2 + \tan^3 x} + C \end{aligned}$$

$$58. \int \frac{\sin \sqrt{\theta}}{\sqrt{\theta} \cos^3 \sqrt{\theta}} d\theta = \int \frac{\sin \sqrt{\theta}}{(\sqrt{\theta}) (\sqrt{\cos^3 \sqrt{\theta}})} d\theta$$

$$\text{Let } u = \cos \sqrt{\theta}, \quad du = -\sin \sqrt{\theta} \cdot \frac{1}{2\sqrt{\theta}} d\theta$$

$$\begin{aligned} \int \frac{\sin \sqrt{\theta}}{(\sqrt{\theta}) (\sqrt{\cos^3 \sqrt{\theta}})} d\theta &= \int \frac{-2}{\sqrt{u^3}} du \\ &= \int -2u^{-3/2} du \\ &= 4u^{-1/2} + C \\ &= \frac{4}{\sqrt{u}} + C \\ &= \frac{4}{\sqrt{\cos \sqrt{\theta}}} + C \end{aligned}$$

$$59. \frac{ds}{dt} = 12t(3t^2 - 1)^3, \quad s(1) = 3$$

$$\begin{aligned} s(t) &= \int 12t(3t^2 - 1)^3 dt \\ &= \int 2u^3 du \quad (u = 3t^2 - 1, \quad du = 6t dt) \\ &= \frac{1}{2}u^4 + C \\ &= \frac{1}{2}(3t^2 - 1)^4 + C \end{aligned}$$

We now use the initial condition $s(1) = 3$ to find the value of C .

$$\begin{aligned} 3 &= \frac{1}{2}(3(1)^2 - 1)^4 + C &\Leftrightarrow & 3 = \frac{1}{2} \cdot 2^4 + C \\ & &\Leftrightarrow & 3 = 8 + C \\ & &\Leftrightarrow & -5 = C \end{aligned}$$

Therefore we get:

$$s(t) = \frac{1}{2}(3t^2 - 1)^4 - 5$$

62. $\frac{dr}{d\theta} = 3 \cos^2 \left(\frac{\pi}{4} - \theta \right)$, $r(0) = \frac{\pi}{8}$

$$\begin{aligned} r(\theta) &= \int 3 \cos^2 \left(\frac{\pi}{4} - \theta \right) d\theta \\ &= \int 3 \left[\frac{1}{2} + \frac{1}{2} \cos \left(\frac{\pi}{2} - 2\theta \right) \right] d\theta \\ &= \int -\frac{3}{2} \left[\frac{1}{2} + \frac{1}{2} \cos u \right] du \quad (u = \frac{\pi}{2} - 2\theta, \quad du = -2 d\theta) \\ &= -\frac{3}{2} \left[\frac{u}{2} + \frac{1}{2} \sin u \right] + C \\ &= -\frac{3}{2} \left[\frac{\pi/2 - 2\theta}{2} + \frac{1}{2} \sin \left(\frac{\pi}{2} - 2\theta \right) \right] + C \end{aligned}$$

We now use the initial condition $r(0) = \frac{\pi}{8}$ to find the value of C .

$$\begin{aligned} \frac{\pi}{8} &= -\frac{3}{2} \left[\frac{\pi/2}{2} + \frac{1}{2} \sin \left(\frac{\pi}{2} \right) \right] + C && \Leftrightarrow \quad \frac{\pi}{8} = -\frac{3}{2} \left[\frac{\pi}{4} + \frac{1}{2} \right] + C \\ &&& \Leftrightarrow \quad \frac{\pi}{8} = -\frac{3\pi}{8} - \frac{3}{4} + C \\ &&& \Leftrightarrow \quad \frac{\pi}{2} + \frac{3}{4} = C \end{aligned}$$

Therefore we get:

$$\begin{aligned} r(\theta) &= -\frac{3}{2} \left[\frac{\pi/2 - 2\theta}{2} + \frac{1}{2} \sin \left(\frac{\pi}{2} - 2\theta \right) \right] + \frac{\pi}{2} + \frac{3}{4} \\ &= -\frac{3\pi}{8} + \frac{3\theta}{2} - \frac{3}{4} \sin \left(\frac{\pi}{2} - 2\theta \right) + \frac{\pi}{2} + \frac{3}{4} \\ &= \frac{\pi}{8} + \frac{3\theta}{2} - \frac{3}{4} \sin \left(\frac{\pi}{2} - 2\theta \right) + \frac{3}{4} \\ &= \frac{\pi}{8} + \frac{3\theta}{2} - \frac{3}{4} \cos(2\theta) + \frac{3}{4} \end{aligned}$$

63. $\frac{d^2s}{dt^2} = -4 \sin \left(2t - \frac{\pi}{2} \right)$, $s'(0) = 100$, $s(0) = 0$

$$\begin{aligned} s'(t) &= \int -4 \sin \left(2t - \frac{\pi}{2} \right) dt \\ &= \int -2 \sin u du \quad (u = 2t - \frac{\pi}{2}, \quad du = 2 dt) \\ &= 2 \cos u + C_1 \\ &= 2 \cos \left(2t - \frac{\pi}{2} \right) + C_1 \end{aligned}$$

We use the initial condition $s'(0) = 100$ to find the value of C_1 .

$$100 = 2 \cos\left(-\frac{\pi}{2}\right) + C_1 \quad \Leftrightarrow \quad 100 = C_1$$

Therefore we get:

$$s'(t) = 2 \cos\left(2t - \frac{\pi}{2}\right) + 100$$

Now we do the whole process again to find $s(t)$.

$$\begin{aligned} s(t) &= \int \left[2 \cos\left(2t - \frac{\pi}{2}\right) + 100 \right] dt \\ &= \int [\cos u + 50] du \quad \left(u = 2t - \frac{\pi}{2}, \quad du = 2 dt \right) \\ &= \sin u + 50u + C_2 \\ &= \sin\left(2t - \frac{\pi}{2}\right) + 50\left(2t - \frac{\pi}{2}\right) + C_2 \end{aligned}$$

We use the initial condition $s(0) = 0$ to find the value of C_2 .

$$\begin{aligned} 0 &= \sin\left(-\frac{\pi}{2}\right) + 50\left(-\frac{\pi}{2}\right) + C_2 \quad \Leftrightarrow \quad 0 = -1 - 25\pi + C_2 \\ &\Leftrightarrow \quad 1 + 25\pi = C_2 \end{aligned}$$

Therefore we get:

$$\begin{aligned} s(t) &= \sin\left(2t - \frac{\pi}{2}\right) + 50\left(2t - \frac{\pi}{2}\right) + 1 + 25\pi \\ &= \sin\left(2t - \frac{\pi}{2}\right) + 100t - 25\pi + 1 + 25\pi \\ &= \sin\left(2t - \frac{\pi}{2}\right) + 100t + 1 \end{aligned}$$

67. All three integrals are correct. To see how this is possible, recall your various trig identities.

$$(a) = (b)$$

$$\begin{aligned} \sin^2 x + C_1 &= 1 - \cos^2 x + C_1 \\ &= -\cos^2 x + C_2 \quad (1 \text{ can be absorbed into the constant term } C.) \end{aligned}$$

(b) = (c)

$$\begin{aligned} -\cos^2 x + C_2 &= -\left(\frac{1}{2} + \frac{1}{2}\cos(2x)\right) + C_2 \\ &= -\frac{1}{2}\cos(2x) + C_2 \quad \left(-\frac{1}{2} \text{ can be absorbed into the constant term } C.\right) \end{aligned}$$