HW #6 Solutions

8.5.37) $\int \underbrace{x}\underbrace{\sec^2 x}\, dx = x\tan x - \int \tan x\, dx$

$= \boxed{x\tan x + \ln|\cos x| + C}$

38) $\int \underbrace{\theta}\underbrace{\sec\theta\tan\theta}\, d\theta = \theta\sec\theta - \int \sec\theta\, d\theta$

$\boxed{= \theta\sec\theta - \ln|\sec\theta + \tan\theta| + C}$

44) $\int_0^{\pi/8} \sin 2x \cos 2x\, \underline{dx} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \sin y \cos y\, dy$

$= \frac{1}{2}\int_0^{\sin(\pi/4)} u\, \cancel{du} = \frac{1}{4}\sin^2(\pi/4) = \frac{1}{4}\left(\frac{1}{\sqrt{2}}\right)^2 = \boxed{\frac{1}{8}}$

$u = \sin y,\ du = \cos y\, dy$

45) $\int_0^1 \tan(1-x)\, dx = -\int_1^0 \tan y\, dy = \int_0^1 \tan y\, dy$

$= -\ln|\cos y|\Big|_0^1 = \boxed{-\ln(\cos(1))}$

52) $\int_0^{\pi} x\sin x\, dx = -x\cos x\Big|_0^{\pi} + \int_0^{\pi} \cos x\, dx$

$= \pi + (\sin\pi - \sin 0) = \boxed{\pi}$

6.3.15) $\int \frac{-2}{x^2-16}\, dx = -2\int \frac{dx}{(x-4)(x+4)}$

$\frac{A}{x+4} + \frac{B}{x-4} = \frac{1}{(x+4)(x-4)} \iff A(x-4) + B(x+4) = 1$

$\iff x(A+B) + 4(B-A) = 1 \iff \begin{cases} A+B=0 \\ B-A=\frac{1}{4} \end{cases} \iff \begin{cases} A=-B \\ 2B=\frac{1}{4} \end{cases}$

$\iff B = \frac{1}{8},\ A = -\frac{1}{8}$

$\therefore \int \frac{-2}{x^2-16}\, dx = \frac{1}{4}\int \frac{dx}{x+4} - \frac{1}{4}\int \frac{dx}{x-4} = \boxed{\frac{1}{4}\ln\left|\frac{x+4}{x-4}\right| + C}$

18) $\int \frac{3}{x^2-3x}dx = 3\int \frac{dx}{x(x-3)}$

$A(x-3) + Bx = 1 \Leftrightarrow x(A+B) - 3A = 1 \Leftrightarrow A = \frac{-1}{3}, B = -A = \frac{1}{3}$

$= \int\left(\frac{-1}{x} + \frac{1}{x-3}\right)dx = \boxed{\ln\left|\frac{x-3}{x}\right| + C}$

23) $\int \frac{5-x}{2x^2+x-1}dx = \int \frac{5-x}{(2x-1)(x+1)}dx$

$\frac{A}{2x-1} + \frac{B}{x+1} = \frac{5-x}{(2x-1)(x+1)} \Leftrightarrow A(x+1) + B(2x-1) = 5-x$

$\Leftrightarrow x(A+2B) + (A-B) = 5-x \Leftrightarrow \begin{cases} A+2B=-1 \\ A-B=5 \end{cases}$

$\Leftrightarrow \begin{cases} A=-1-2B \\ -1-2B-B=5 \end{cases} \Leftrightarrow \begin{cases} A=-1-2B=-1+4=3 \\ B=-2 \end{cases}$

$= \int\left(\frac{3}{2x-1} - \frac{2}{x+1}\right)dx = \boxed{\frac{3}{2}\ln|x-\tfrac{1}{2}| - 2\ln|x+1| + C}$

24 ~~26)~~ $\cancel{\frac{x+1}{x^2+4x+3} = \frac{x+1}{(x+3)(x+1)} \neq \frac{A}{x+3} + \frac{B}{x+1} \Leftrightarrow A(x+1) + B(x+3) = x+1}$

$\cancel{\Leftrightarrow x(A+B) + (A+3B) = x+1 \Leftrightarrow \begin{cases} A+B=1 \\ A+3B=1 \end{cases}}$

26) $\frac{3x^2-7x-2}{x^3-x} = \frac{3x^2-7x-2}{x(x^2-1)} = \frac{3x^2-7x-2}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$

$\Leftrightarrow A(x-1)(x+1) + Bx(x+1) + Cx(x-1) = 3x^2-7x-2$

$\Leftrightarrow A(x^2-1) + B(x^2+x) + C(x^2-x) = 3x^2-7x-2$

$\Leftrightarrow x^2(A+B+C) + x(B-C) + (-A) = 3x^2-7x-2$

$\Leftrightarrow \begin{cases} A+B+C=3 \\ B-C=-7 \\ -A=-2 \end{cases} \Leftrightarrow \begin{cases} B+C=1 \\ B-C=-7 \\ A=2 \end{cases} \Leftrightarrow \begin{cases} C=1-B=4 \\ B=\frac{1}{2}(1+(-7))=-3 \\ A= \end{cases}$

$\therefore \int \frac{3x^2-7x-2}{x^3-x}dx = \int\left(\frac{2}{x} - \frac{3}{x-1} + \frac{4}{x+1}\right)dx$

$= \boxed{2\ln|x| - 3\ln|x-1| + 4\ln|x+1| + C}$

30) $\dfrac{x^4}{(x-1)^3} = \dfrac{x^4}{(x-1)(x^2-2x+1)} = \dfrac{x^4}{x^3-3x^2+3x-1}$

$= x + \dfrac{x^4 - x^4 + 3x^3 - 3x^2 + x}{x^3-3x^2+3x-1} = x+3 + \dfrac{-3x^2+x+9x^2-9x+3}{x^3-3x^2+3x-1}$

$= x+3+\dfrac{6x^2-8x+3}{(x-1)^3}$

$\dfrac{6x^2-8x+3}{(x-1)^3} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x-1)^3}$

$\Leftrightarrow 6x^2-8x+3 = A(x-1)^2 + B(x-1) + C$

$\Leftrightarrow 6x^2-8x+3 = x^2(A) + x(-2A+B) + (A-B+C)$

$\Leftrightarrow \begin{cases} A=6 \\ -2A+B=-8 \\ A-B+C=3 \end{cases} \Leftrightarrow \begin{cases} A=6 \\ B=-8+12=4 \\ C=3-6+4=1 \end{cases}$

$\therefore \displaystyle\int \frac{x^4}{(x-1)^3}\,dx = \int\left(\frac{6}{x-1} + \frac{4}{(x-1)^2} + \frac{1}{(x-1)^3} + x+3\right)dx$

$= \boxed{6\ln|x-1| - \dfrac{4}{x-1} - \dfrac{1}{2(x-1)^2} + \dfrac{1}{2}x^2 + 3x + C}$

31) $\dfrac{3x^2+3x+1}{x(x+1)^2} = \dfrac{A}{x} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$

$\Leftrightarrow 3x^2+3x+1 = A(x+1)^2 + Bx(x+1) + Cx$

$\Leftrightarrow 3x^2+3x+1 = x^2(A+B) + x(2A+B+C) + (A)$

$\Leftrightarrow \begin{cases} A=1 \\ 2A+B+C=3 \\ A+B=3 \end{cases} \Leftrightarrow \begin{cases} A=1 \\ C=3-2-2=-1 \\ B=3-1=2 \end{cases}$

$\therefore \displaystyle\int \frac{3x^2+3x+1}{x(x+1)^2}\,dx = \int\left(\frac{1}{x} + \frac{2}{x+1} - \frac{1}{(x+1)^2}\right)dx$

$= \boxed{\ln|x| + 2\ln|x+1| + \dfrac{1}{x+1} + C}$

38) $\dfrac{x^3-1}{x^2-4} = x + \dfrac{x^3-1-x^3+4x}{x^2-4} = x + \dfrac{4x-1}{x^2-4}$

$\dfrac{4x-1}{x^2-4} = \dfrac{4x-1}{(x-2)(x+2)} = \dfrac{A}{x-2} + \dfrac{B}{x+2} \Leftrightarrow A(x+2)+B(x-2) = 4x-1$

$\Leftrightarrow \begin{cases} A+B=4 \\ 2A-2B=-1 \end{cases} \Leftrightarrow \begin{cases} A+B=4 \\ A-B=-\frac{1}{2} \end{cases} \Leftrightarrow \begin{cases} A=\frac{1}{2}(4+(-\frac{1}{2})) = \frac{7}{4} \\ B = \frac{1}{2}+\frac{7}{4} = \frac{9}{4} \end{cases}$

$\therefore \int_0^1 \dfrac{x^3-1}{x^2-4}\,dx = \frac{7}{4}\int_0^1 \dfrac{dx}{x-2} + \frac{9}{4}\int_0^1 \dfrac{dx}{x+2} + \int x\,dx$

$= \left[\frac{7}{4}\ln|x-2| + \frac{9}{4}\ln|x+2|\right]\Big|_0^1 + \frac{1}{2}$

$= \left[\frac{7}{4}\ln(1) + \frac{9}{4}\ln(3) - \frac{7}{4}\ln(2) - \frac{9}{4}\ln(2)\right] + \frac{1}{2}$

$= \boxed{\frac{9}{4}\ln(3) - 4\ln(2) + \frac{1}{2}}$

39) $\dfrac{x^3-4x^2-3x+3}{x^2-3x} = x + \dfrac{-x^2-3x+3}{x^2-3x} = x-1 + \dfrac{-6x+3}{x^2-3x}$

$\dfrac{-6x+3}{x^2-3x} = \dfrac{-6x+3}{x(x-3)} = \dfrac{A}{x} + \dfrac{B}{x-3} \Leftrightarrow A(x-3)+Bx = -6x+3$

$\Leftrightarrow \begin{cases} -3A=3 \\ A+B=-6 \end{cases} \Leftrightarrow \begin{cases} A=-1 \\ B=-6-(-1)=-5 \end{cases}$

$\therefore \int_1^2 \;dx = \int_1^2 \left(x-1-\dfrac{1}{x}-\dfrac{5}{x-3}\right)dx$

$= \left[\frac{1}{2}x^2 - x - \ln|x| - 5\ln|x-3|\right]\Big|_1^2$

$= \left[(2-2-\ln(2)-5\ln(1)) - (\frac{1}{2}-1-\ln(1)-5\ln(2))\right]$

$= (-\ln(2)) - (-5\ln(2)-\frac{1}{2}) = \boxed{4\ln(2)+\frac{1}{2}}$

43) $\frac{x+1}{x^2-x} = \frac{x+1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} \Leftrightarrow A(x-1)+Bx = x+1$

$\Leftrightarrow \begin{cases} A+B=1 \\ -A=1 \end{cases} \Leftrightarrow \begin{cases} B=1-(-1)=2 \\ A=-1 \end{cases}$

$\therefore \int_2^5 \frac{x+1}{x^2-x}\,dx = \int_2^5 \left(\frac{-1}{x} + \frac{2}{x-1}\right)dx = \left[-\ln|x| + 2\ln|x-1|\right]\Big|_2^5$

$= -\ln 5 + 2\ln 4 + \ln 2 - 2\ln 1$

$= \boxed{5\ln 2 - \ln 5}$

44) $\frac{x^2+2x-1}{x^2-4} = 1 + \frac{2x+3}{x^2-4}$

$\frac{2x+3}{x^2-4} = \frac{2x+3}{(x+2)(x-2)} = \frac{A}{x+2} + \frac{B}{x-2} \Leftrightarrow A(x-2)+B(x+2) = 2x+3$

$\Leftrightarrow \begin{cases} A+B=2 \\ -2A+2B=3 \end{cases} \Leftrightarrow \begin{cases} B+A=2 \\ B-A=\frac{3}{2} \end{cases} \Leftrightarrow \begin{cases} B=\frac{1}{2}(2+\frac{3}{2})=\frac{7}{4} \\ A=\frac{7}{4}-\frac{3}{2}=\frac{1}{4} \end{cases}$

$\frac{x^2+2x-1}{x^2-4} = 0 \Leftrightarrow x^2+2x-1=0$

$\Leftrightarrow x = \frac{-2 \pm \sqrt{4+4}}{2} = -1 \pm \frac{1}{2}\sqrt{8} = -1 \pm \sqrt{2}$

$\int_{-1}^{-1+\sqrt{2}} \frac{x^2+2x-1}{x^2-4}\,dx = \int_{-1}^{-1+\sqrt{2}} \left(1 + \frac{1}{4}\frac{1}{x+2} + \frac{7}{4}\frac{1}{x-2}\right)dx$

$= \left[x + \frac{1}{4}\ln|x+2| + \frac{7}{4}\ln|x-2|\right]\Big|_{-1}^{-1+\sqrt{2}}$

$= \left[\left(-1+\sqrt{2} + \frac{1}{4}\ln(1+\sqrt{2}) + \frac{7}{4}\ln(3-\sqrt{2})\right) - \left(-1 + \frac{1}{4}\ln(1) + \frac{7}{4}\ln(3)\right)\right]$

$= \boxed{\sqrt{2} + \frac{1}{4}\ln(1+\sqrt{2}) + \frac{7}{4}\ln(9+3\sqrt{2})}$