Announcements

- Course Webpage
- Syllabus
- WebAssign Course Key: ucdavis 0867 6634
  - Create math account: need CRN 30099
- Office hours W 2-4pm Wellman 115
- HW1 Due 1/11

Notation

- $x, y, z, t$: (usually) variables representing real numbers
- $n, i$: (usually) variables representing integers
- $a, b, c, r, p$: (usually) constants
- $\epsilon, \delta$: (usually) a variable representing a small real number
- $A \rightarrow B$: A “then” $B$ OR $t \rightarrow 0$: $t$ “approaches” 0
- $\in$: is an element of
- $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$: the set of all integers, rational numbers, real numbers
- $I$: the interval $[0, 1]$, ie, the set of all real numbers between 0,1

4 Exponential and Logarithmic Functions

4.1 Exponential Functions

4.2 Natural Exponential Functions

**Definition.** (Exponential Function) If $a > 0$ and $a \neq 1$ then the *exponential function* with base $a$ is given by $f(x) = a^x$.

Properties of Exponents

Let $a$ and $b$ be positive real numbers.

1. $a^0 = 1$
2. $a^x a^y = a^{x+y}$
3. $\frac{a^x}{a^y} = a^{x-y}$
4. $(a^x)^y = a^{xy}$
5. $(ab)^x = a^x b^x$
6. $(\frac{a}{b})^x = \frac{a^x}{b^x}$
7. $a^{-x} = \frac{1}{a^x}$
**Definition.** (Natural Exponential Function) The irrational number $e$ is defined to be

$$e = \lim_{x \to 0} (1 + x)^{1/x}$$

The natural exponential function is

$$f(x) = e^x \quad \left( = \lim_{n \to \infty} \left( 1 + \frac{x}{n} \right)^n = \exp(x) \right)$$

**Applications**

A. Modeling a population To model the growth of a population an exponential function is often used. One way to model the growth of a populations whose quantity is restricted is called the logistic growth model

$$f(t) = \frac{a}{1 + be^{-kt}}, \quad t \geq 0$$

B. Compounding interest

An amount of $P$, the principal, is deposited into an account at an annual interest rate of $r$. The balance $A_n$ after $t$ years is

$$A_n = P \left( 1 + \frac{r}{n} \right)^{nt}$$

where $n$ is the number of compoundings in a year. The limit of $A_n$ as $n \to \infty$ is the balance $A$ after $t$ years of continuous compounding, or

$$A = \lim_{n \to \infty} A_n = \lim_{n \to \infty} P \left( 1 + \frac{r}{n} \right)^{nt} = Pe^{rt}$$

C. The annual interest rate $r$ is called the *nominal rate* and is different from the actual percentage of interest earned after 1 year. This actual rate at which interest is earned is the effective rate

$$r_{eff} = \left( 1 + \frac{r}{n} \right)^n - 1$$

D. The *present value* of $A_n$ finds the principal $P$ needed to obtain $A_n$ given a fixed rate of interest $r$, $t$ years from now.

**4.3 Derivative of Exponential Functions**

Let $u$ be a differentiable function of $x$.

$$\frac{d}{dx} [e^x] = e^x \quad \frac{d}{dx} [e^u] = e^u \frac{du}{dx}$$