Announcements

- W 2-4p Wellman 115

Probability Theory

**Definition.** An *experiment* is any procedure that can be infinitely repeated. The results of the procedure are called *outcomes*. The set of all outcomes is called a *sample space*.

A mathematical description of an experiment consists of three parts:

1. A set of *events* $\mathcal{F}$, where each event is a set containing zero or more outcomes.
2. A sample space $S$.
3. The assignment of probabilities to the events—i.e., a function $P$ mapping from events to probabilities.

Probability is a number between 0 and 1. That is, $P : \mathcal{F} \rightarrow [0, 1]$.

A *random variable* $X : S \rightarrow R$ is a function that assigns a number to every outcome. When the range $R$ (or image) of $X$ is finite (or countably infinite), $X$ is called a *discrete random variable*. If the range $R$ is (uncountably) infinite, $X$ is called a *continuous random variable*.

The collection of probabilities corresponding to the values of the random variable is called a *probability distribution* of the random variable.

A *probability density function* $f$ of a random variable $X$ is a function which is non-negative, continuous on the range $R$ of $X$, and $\int_R f(x) \, dx = 1$. It used to specify a random variable falling within a particular range of values, as opposed to taking on any one value. Not all random variables have probability density functions—discrete random variables do not.

If the range of a discrete random variable consists of $m$ different values $\{x_1, \ldots, x_m\}$, then the *expected value* or *mean*, of the random variable is

$$\mu = \mathbb{E}[X] = x_1 P(x_1) + x_2 P(x_2) + \cdots + x_m P(x_m)$$

The variance of $X$ is

$$\sigma^2 = \text{Var}(X) = (x_1 - \mu)^2 P(x_1) + (x_2 - \mu)^2 P(x_2) + \cdots + (x_m - \mu)^2 P(x_m)$$

The standard deviation of $X$ is

$$\sigma = \sqrt{\text{Var}(X)}$$

If $f$ is the probability density of a continuous random variable $X$ whose range is $R$, then the expected value is

$$\mathbb{E}[X] = \int_R x \, f(x) \, dx$$

The variance is

$$\sigma^2 = \text{Var}(X) = \int_R (x - \mu)^2 f(x) \, dx$$

The standard deviation is

$$\sigma = \sqrt{\text{Var}(X)}$$