Take this test as you would take a test in class. When you are done, check your work against the answers given in the back of the book.

1. Four students answer a true-false question on an exam. The random variable $x$ is the number of answers of true among the four students.
   (a) Write the sample space for the possible outcomes.
   (b) Find the frequency distribution for the random variable $x$.
   (c) Find the probability distribution for the random variable $x$.

2. A card is chosen at random from a standard 52-card deck of playing cards. What is the probability that the card will be red and not a face card?

In Exercises 3 and 4, sketch a graph of the probability distribution and find the indicated probabilities.

3.  
   \[
   \begin{array}{c|cccc}
   x & 1 & 2 & 3 & 4 \\
   \hline
   P(x) & \frac{3}{16} & \frac{2}{16} & \frac{5}{16} & \frac{5}{16} \\
   \end{array}
   \]
   (a) $P(x < 3)$  
   (b) $P(x \geq 3)$

4.  
   \[
   \begin{array}{c|cccccc}
   x & 7 & 8 & 9 & 10 & 11 \\
   \hline
   P(x) & 0.21 & 0.13 & 0.19 & 0.42 & 0.05 \\
   \end{array}
   \]
   (a) $P(7 \leq x \leq 10)$  
   (b) $P(x > 8)$

In Exercises 5 and 6, find the expected value, variance, and standard deviation for the given probability distribution.

5.  
   \[
   \begin{array}{c|cccc}
   x & 0 & 1 & 2 & 3 \\
   \hline
   P(x) & \frac{2}{10} & \frac{4}{10} & \frac{3}{10} & \frac{3}{10} \\
   \end{array}
   \]

6.  
   \[
   \begin{array}{c|cccc}
   x & -2 & -1 & 0 & 1 \\
   \hline
   P(x) & 0.141 & 0.305 & 0.257 & 0.063 \\
   \end{array}
   \]
   \[
   \begin{array}{cccc}
   & 2 \\
   \hline
   P(x) & 0.234 \\
   \end{array}
   \]

In Exercises 7–9, use a graphing utility to graph the function. Then determine whether the function $f$ represents a probability density function over the given interval. If $f$ is not a probability density function, identify the condition(s) that is (are) not satisfied.

7. $f(x) = \frac{1}{16}$, $[0, 8]$  
8. $f(x) = \frac{3-x}{6}$, $[-1, 1]$  
9. $f(x) = \frac{3}{4}e^{-3x/4}$, $[0, \infty)$

In Exercises 10–12, find the indicated probabilities for the probability density function.

10. $f(x) = \frac{2x}{9}$, $[0, 3]$  
    (a) $P(0 \leq x \leq 1)$  
    (b) $P(2 \leq x \leq 3)$

11. $f(x) = 4(x - x^3)$, $[0, 1]$  
    (a) $P(0 < x < 0.5)$  
    (b) $P(0.25 < x < 1)$

12. $f(x) = 2xe^{-x^2}$, $[0, \infty)$  
    (a) $P(x < 1)$  
    (b) $P(x \geq 1)$

In Exercises 13–15, find the mean, variance, and standard deviation of the probability density function.

13. $f(x) = \frac{1}{14}$, $[0, 14]$  
14. $f(x) = 3x - \frac{3}{2}x^2$, $[0, 1]$  
15. $f(x) = e^{-x}$, $[0, \infty)$

16. An intelligence quotient or IQ is a number that is meant to measure intelligence. The IQs of students in a school are normally distributed with a mean of 110 and a standard deviation of 10. Use a symbolic integration utility to find the probability that a student selected at random will have an IQ within one standard deviation of the mean.