Exercise 1. Given integers $a, b, c$ with $a < b$ and $c < 0$, show the following:

(a) $a + c < b + c$
(b) $a^2 \geq 0$
(c) $ac > bc$
(d) $c^3 < 0$

Exercise 2. Prove by induction that $2^n < n!$ for all $n \geq 4$.

Exercise 3. Find the mistake in the following proof by induction, which shows that all horses are the same color. More specifically, it shows that for every positive integer $n$, in any group of $n$ horses, all the horses have the same color. The base case is $n = 1$, which is obvious. Now, suppose we know the statement for groups of size $n$, and we have a group of $n + 1$ horses. If we remove one horse from the group, we have a group of size $n$, so by the induction hypothesis, all the horses in that group have the same color. If we put the removed horse back into the group, and remove a different horse, we can again apply the induction hypothesis to conclude that all $n$ horses have the same color. But this means that all $n + 1$ horses in the group must have had the same color, completing the proof.

Exercise 4. What can you conclude if $a$ and $b$ are integers such that $a \mid b$ and $b \mid a$?

Exercise 5. Are there integers $a, b, c$ such that $a \mid bc$, but $a \nmid b$, $a \nmid c$?