Exercise 1. Show that for any integer $k$, we have

$$(7k + 5, 2k + 1) = 1 \text{ or } 3.$$ 

Can both possibilities occur for different values of $k$?

Exercise 2. For each pair of integers $a, b$ from each part of problem 6 of Problem Set #2, you used the Euclidean algorithm to find $(a, b)$. Now use the extended Euclidean algorithm to find $x, y \in \mathbb{Z}$ such that $ax + by = (a, b)$.

Exercise 3. Recall that in lecture, we mentioned a modified Euclidean algorithm, where you simply use the modified division algorithm from Problem Set #2 instead of the regular division algorithm. Use this modified Euclidean algorithm to find $(1597, 2584)$.

(Note that these are Fibonacci numbers – the regular Euclidean algorithm would require 15 steps)

Exercise 4. Use trial division to determine which of the following numbers are prime and which are composite, and to give the prime factorizations of the composite ones (you may use the table of primes under 100 given in Figure 3.1 of the book). Show work as follows: for primes, keep a list of which numbers you tried dividing by for each case. For composite numbers, show clearly which numbers you tried dividing by, and what you did each time you found a divisor.

(a) 2477.
(b) 2891.
(c) 4129.
(d) 7181.

Exercise 5. For each $n \geq 1$, describe (in terms of $n$) how many rational roots the polynomial $x^n - 1$ has.

Exercise 6. Show that $\log_{5} 8$ is irrational. More precisely, show that there do not exist positive integers $a, b$ such $5^b = 8^a$. (You should understand why these two statements are the same; we use the second because we have not formally defined either log or irrational.)