For the below problems, be sure to use relevant material relating to Wilson’s theorem and Fermat’s Little Theorem rather than simple brute force calculation.

**Exercise 1.** What is the remainder when $5!25!$ is divided by 31?

**Exercise 2.** What is the remainder when $40!$ is divided by 1763?

**Exercise 3.** What is the remainder when $2^{1,000,000}$ is divided by 17?

**Exercise 4.** Show that if $n > 4$ is a composite integer, then $(n - 1)! \equiv 0 \pmod{n}$.

**Exercise 5.** Show that $a^6 - 1$ is divisible by 168 whenever $(a, 42) = 1$.

**Exercise 6.** Show that if $p, q$ are distinct primes, then

$$p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}.$$  

**Exercise 7.** Show that if $p > 3$ is prime then

$$2^{p-2} + 3^{p-2} + 6^{p-2} \equiv 1 \pmod{p}.$$