Give yourself 50 minutes to take this exam. Be sure to fully justify all your answers.

1 (32 pts.) Determine whether or not the following limits exist, and calculate them. If the limit does not exist as a number, state whether or not it can be written as $\infty$ or $-\infty$.

(a) $\lim_{x \to 3} \frac{x-3}{x^2-2x-3}$

(b) $\lim_{t \to 0} \frac{\sqrt{t+1}-1}{t}$

(c) $\lim_{x \to 1^+} \frac{2-x^2}{1-x}$

(d) $\lim_{x \to \infty} \sin x$
2  (8 pts.) Show that \( \lim_{x \to 0} x \sin(1/x) = 0 \).

3  (12 pts.) In terms of \( \epsilon \) and \( \delta \), show that \( \lim_{x \to -3} x^2 = 9 \).
For each question, answer only “true” or “false”. You will receive 5 points for a correct answer, 2 points for no answer, and 0 points for an incorrect answer. There is no partial credit.

(a) If \( f(x) > g(x) \) on \((0, 1)\), and both \( \lim_{x \to 0^+} f(x) \) and \( \lim_{x \to 0^+} g(x) \) exist, then \( \lim_{x \to 0^+} f(x) > \lim_{x \to 0^+} g(x) \).

(b) If \( \lim_{x \to 0^+} f(x) \) is a positive number, then there is some interval \((0, c)\) for \( c > 0 \) on which \( f(x) \) is positive.

(c) The function \( f(x) = |x|/x \) has a removable discontinuity at \( x = 0 \).

(d) For every function \( f(x) \) such that \( \lim_{x \to 0} f(x) = 0 \) and every \( g(x) \) such that \( \lim_{x \to 0} g(x) = \infty \), the limit \( \lim_{x \to 0} f(x) \cdot g(x) \) does not exist.
Consider the function

\[ f(x) = \begin{cases} 
  \frac{x^3 - 2x^2 + 2x - 4}{x^2 - 3x + 2} : & x \neq 1, 2 \\
  4 : & x = 1, 2 
\end{cases} \]

At which points is this function continuous, and at which points is it discontinuous? For each discontinuity, say whether or not it is removable. (Hint: factor the denominator first) Find all asymptotes to the graph. Sketch the graph.
For the function $f(x) = \sqrt{x}$, what is the average rate of change of $f(x)$ from $x = 3$ to $x = 4$? What is the instaneous rate of change at $x = 3$? (Compute this from our definitions, without using derivative laws)