1 (40 pts.) Differentiate the following functions. Show your work if you wish to receive partial credit. You do not need to simplify your answers.

(a) \( f(x) = \frac{3x+7}{\sqrt{x^4+1}}. \)

Answer:
\[
 f'(x) = \frac{-3x^4 - 14x^3 + 3}{(x^4 + 1)^{3/2}}.
\]

(b) \( f(x) = \frac{x^2+1}{\tan^{-1}(x^2+1)}. \)

Answer:
\[
 f'(x) = \frac{2x((x^4 + 2x^2 + 2)\tan^{-1}(x^2 + 1) - (x^2 + 1))}{(\tan^{-1}(x^2 + 1))^2(x^4 + 2x^2 + 2)}. \]

(c) \( f(x) = e^{2x-1}\sqrt{x + 1}. \)

Answer:
\[
 f'(x) = \frac{e^{2x-1}(4x + 5)}{2\sqrt{x + 1}}.
\]

(d) \( f(x) = \sec^2(x + \sin x). \)

Answer:
\[
 f'(x) = 2(1 + \cos x)\sec^2(x + \sin x)\tan(x + \sin x).
\]

(e) \( f(x) = (\ln x)^{10}. \)

Answer:
\[
 f'(x) = \frac{10(\ln x)^9}{x}.
\]
2 (10 pts.) Suppose $u(x)$ is a differentiable function of $x$, and let $f(x) = x^{u(x)}$, for $x > 0$. Find $f'(x)$, in terms of $x, u(x)$, and $u'(x)$. Hint: take the natural log of both sides and use implicit differentiation.

**Answer:**

$$f'(x) = x^{u(x)} (u'(x) \ln x + \frac{u(x)}{x}).$$

3 (20 pts.) Consider the parametric curve $(-\sqrt{t + 1}, \sqrt{3t})$.

(a) Find the tangent line to the curve at the point corresponding to $t = 3$.

**Answer:** $y = -2x - 1$.

(b) Find $d^2y/(dx)^2$ at the same point.

**Answer:** $d^2y/(dx)^2 = -1/3$ at $t = 3$.

4 (10 pts.) Find the equation of the line tangent to the curve defined implicitly by $2 \sin^{-1} y = x^2$ at the point $(\sqrt{\pi}, \sqrt{1/2})$.

**Answer:** $y = \frac{\sqrt{\pi}}{x} x + \frac{2-\pi}{2\sqrt{2}}$.

5 (20 pts.) Consider the function

$$f(x) = \begin{cases} x \sin \frac{1}{x} : & x \neq 0 \\ 0 : & x = 0 \end{cases}.$$

(a) Is $f(x)$ continuous at $x = 0$? Why or why not?

**Answer:** Yes. $x \sin \frac{1}{x}$ is sandwiched by $x$ and $-x$.

(b) Is $f(x)$ differentiable at $x = 0$? Why or why not? If so, what is $f'(0)$?

**Answer:** No. $x \sin \frac{1}{x}$ hits both $y = x$ and $y = -x$ inside $(-\delta, \delta)$ for any $\delta > 0$, so the slope of the secants hits both 1 and $-1$ for any $\delta$. 