Give yourself 50 minutes to take this exam. Be sure to fully justify all your answers.

1 (14 pts.) Determine whether or not the following limits exist, and calculate them. If the limit does not exist as a number, state whether or not it can be written as $\infty$ or $-\infty$.

(a) $\lim_{x \to 1^+} \frac{2-x^2}{1-x}$

(b) $\lim_{x \to \infty} \sin x$
Consider the function

\[ f(x) = \begin{cases} \frac{x^3 - 2x^2 + 2x - 4}{x^2 - 3x + 2} & : x \neq 1, 2 \\ 4 & : x = 1, 2 \end{cases} \]

At which points is this function continuous, and at which points is it discontinuous? For each discontinuity, say whether or not it is removable. Find all asymptotes to the graph. Sketch the graph.
3 (14 pts.) Suppose you are making a square roadsign, with side length $x$. If your customer wants the sign’s area to be within $\epsilon$ of 9 square feet, how close does $x$ have to be to 3 feet?

4 (8 pts.) Does the equation $x^3 - x - 1 = 0$ have a solution in the interval $[1, 2]$? Explain.
5 (20 pts.) For each question, answer only “true” or “false”. There is no partial credit.

(a) If \( f(x) > g(x) \) on \((0, 1)\), and both \( \lim_{x \to 0^+} f(x) \) and \( \lim_{x \to 0^+} g(x) \) exist, then
\[
\lim_{x \to 0^+} f(x) > \lim_{x \to 0^+} g(x).
\]

(b) If \( \lim_{x \to 0^+} f(x) \) is a positive number, then there is some interval \((0, c)\) for \(c > 0\) on which \(f(x)\) is positive.

(c) The function \( f(x) = |x|/x \) has a removable discontinuity at \( x = 0 \).

(d) For every function \( f(x) \) such that \( \lim_{x \to 0} f(x) = 0 \) and every \( g(x) \) such that \( \lim_{x \to 0} g(x) = \infty \), the limit \( \lim_{x \to 0} f(x) \cdot g(x) \) does not exist.
6 (10 pts.) If \( f(x) = \frac{1}{x} \), find \( f'(2) \) directly from the definition of the derivative.

7 (14 pts.) Suppose \( f(x) = x^3 - ax + b \). Find \( a, b \) if \((1, 1)\) is on the graph \( y = f(x) \), and the tangent line to the graph at \((1, 1)\) has slope \(-3\).