Math 21M Brian Osserman Practice Exam 3

Give yourself 1 hour to take this exam. Be sure to fully justify all your answers.

1 (30 pts.) Evaluate the following integrals (consider them as improper integrals where necessary, and state clearly when you are doing so):

(a) $\displaystyle\int_2^3 \frac{x^2}{(x-1)(x^2+2x+1)}\,dx.$

(b) $\displaystyle\int \frac{7\,dx}{(9x^2+1)^2}.$

(c) $\displaystyle\int_0^2 \frac{x+1}{\sqrt{4-x^2}}\,dx.$

2 (20 pts.) The region enclosed by the curves

$$y = x^{1/2}, \quad y = x^{1/4}, \quad 0 \leq x \leq 1$$

is revolved about the y-axis. Find the volume of the resulting solid.

3 (20 pts.) The shape of a water tank is described by revolving the curve $y = x^2$, $0 \leq x \leq 4$, about the y-axis (x, y are in meters). The tank is filled with water. What is the work required to pump all the water out over the top edge of the tank? (The weight-density of water is $10,000$ N/m^3)

4 (10 pts.) Does the following improper integral converge or diverge?

$$\int_2^\infty \frac{x\,dx}{\sqrt{x^4-1}}.$$

5 (10 pts.) Find the general solution of the following differential equation:

$$(\sec x)\frac{dy}{dx} = e^{y+\sin x}.$$

6 (10 pts.) Use the Max-Min inequality to find upper and lower bounds for $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx$. Then do the same thing, but first breaking up the integral as

$$\int_0^1 \frac{1}{1+x^2}\,dx = \int_0^{1/2} \frac{1}{1+x^2}\,dx + \int_{1/2}^1 \frac{1}{1+x^2}\,dx.$$

Can you relate what you've done to Riemann sums?