Exercise 1. Let $Y$ be a projective variety, and $X$ the affine cone over $Y$.

(a) Show that $\dim \mathbb{P}^n_k = n$.
(b) Show that $\text{codim}_{\mathbb{A}^{n+1}_k} X = \text{codim}_{\mathbb{P}^n_k} Y$, and $\dim X = \dim Y + 1$.
(c) Conclude that if $\text{codim}_{\mathbb{P}^n_k} Y = 1$, then $Y = Z_h(F)$ for some homogeneous polynomial $F$.

Exercise 2. (a) Show that if $\varphi : X \to Y$ is a morphism of varieties, and $U \subseteq X$ is an open subset such that the composition $U \to Y$ is an isomorphism, then $U = X$.
(b) Show that if $\varphi : X \to Y$ is a morphism of varieties, and $U \subseteq X$ is an open subset such that $\varphi : U \to Y$ is an isomorphism onto an open subset $V \subseteq Y$, then $\varphi^{-1}(V) = U$.
(c) Give an example to demonstrate that this is false if $X$ is allowed to be an arbitrary prevariety.

Exercise 3. Assume that $k$ doesn’t have characteristic 2, and consider the projective curve $X = Z(X_1^2X_2 - X_0^3 + X_0X_2^2)$.

(a) Show that $X$ is nonsingular.
(b) Show that $X$ is not birational to $\mathbb{P}^1_k$.

Exercise 4. Let $\varphi : X \to Y$ be a morphism, with $Y$ a variety and $X$ a prevariety.

(a) Show that if $P, Q \in X$ have $(P, Q)$ in the closure of $\Delta(X)$, then $\varphi(P) = \varphi(Q)$.
(b) Suppose that there is an open cover $\{V_i\}$ of $Y$ such that each $V_i$ is isomorphic to an affine variety, and also each $\varphi^{-1}(V_i)$ is isomorphic to an affine variety. Show that $X$ is a variety.