Exercise 1. A topological space $X$ is compact if and only if for every topological space $Y$, the projection map $X \times Y \to Y$ is a closed map.

Hint: given an open cover $\{U_i\}$ of $X$, define $Y$ by adding a single point $\omega$ to $X$, with the topology whose open subsets consist of: any subset of $X$; and any set of the form $\{\omega\} \cup (X \setminus U)$, where $U$ is contained in a finite union of sets $U_i$. Show that the $U_i$ must contain a finite subcover.

Exercise 2. One could define a prevariety $X$ to be universally closed if for all prevarieties $Y$, the projection map $X \times Y \to Y$ is closed. Thus, if $X$ is universally closed and is also a variety, then $X$ is complete. Show that conversely, if $X$ is a complete variety, then $X$ is universally closed. Show the stronger statement that a prevariety $X$ is universally closed if and only if $X \times \mathbb{A}_k^n \to \mathbb{A}_k^n$ is closed for all $n$.

Exercise 3. Do Exercise 6.3 of Chapter I of Hartshorne.

Exercise 4. Do Exercise 6.4 of Chapter I of Hartshorne.