Exercise 1. Let $X, Y$ be nonsingular curves, and $\varphi : X \to Y$ a nonconstant morphism. For $P \in X$, show that $\varphi$ is unramified at $P$ if and only if the induced map $T_P(X) \to T_{\varphi(P)}(Y)$ is injective.

Exercise 2. Let $X \subseteq \mathbb{P}^n_k$ be a nonsingular projective variety, and $P \in \mathbb{P}^n_k \setminus X$. Then the linear projection from $P$ defines a morphism $\varphi : X \to \mathbb{P}^{n-1}_k$.

(a) In terms of the geometry of $X$ in $\mathbb{P}^n_k$, describe which points $Q$ have the property that the induced map $T_Q(X) \to T_{\varphi(Q)}\mathbb{P}^{n-1}_k$ is injective.

(b) Assuming that $\varphi$ is nondegenerate and $X$ is a curve, describe the same set of points as in (a) in terms of the family of divisors on $X$ corresponding to $\varphi$.

Exercise 3. Given $X \subseteq \mathbb{P}^2_k$ a nonsingular projective plane curve, and $P \in \mathbb{P}^2_k \setminus X$, the linear projection from $P$ defines a morphism $\varphi : X \to \mathbb{P}^1_k$.

(a) In terms of plane geometry, describe the ramification points of $\varphi$ and their ramification indices.

(b) If $P = (0, 1, 0)$, describe the ramification points of $\varphi$ in terms of a suitable partial derivative.

(c) (Extra credit) Where possible, extend (b) to a formula for ramification indices (be careful with characteristic! You may find the definition given in Exercise 5.4 of Hartshorne to be helpful).