Exercise 1.  (a) Show that if $f \in A_n$ is a nonzero polynomial, then $Z(f) \neq A^n$.
(b) Without using the ideal-variety correspondence, show that $A^n$ is irreducible for all $n$.

Exercise 2.  Show that the Zariski topology on $A^2_k$ is not equal to the product topology, identifying the underlying sets $A^2_k \cong k^2 \cong A^1_k \times A^1_k$.

Exercise 3.  Consider $Z(x^2 - yz, xz - x) \subseteq A^3_k$. Find the irreducible components, and the corresponding prime ideals of $k[x, y, z]$.

Exercise 4.  All of our definitions so far make perfect sense even if $k$ is not algebraically closed, and most of our basic results still hold. In fact, the only algebra theorem we have used which requires algebraic closure is the Nullstellensatz.
(a) Over your favorite non-algebraically closed field, give a counterexample to the Nullstellensatz.
(b) Give an example of a non-algebraically closed field $k$ and an irreducible polynomial $f \in k[x, y]$ such that $Z(f)$ is nonempty but not irreducible (hint: there are various ways to do this, but one approach is to find an irreducible complex polynomial in two variables which is not a scalar times a real polynomial, and whose real zeroes consist of more than one (but finitely many) points).