Exercise 1. Let $X \subseteq \mathbb{A}_k^n$ be an affine algebraic set, with $f_1, \ldots, f_m$ generating $I(X)$. Then given a point $P \in X$, and a vector $v \in k^n$, the following are equivalent:

(a) $v$ is a tangent vector to $X$ at $P$;
(b) we have
\[ (\partial f_i/\partial x_1(P), \ldots, \partial f_i/\partial x_n(P)) \cdot v = 0 \]
for $i = 1, \ldots, m$;
(c) we have
\[ (\partial f/\partial x_1(P), \ldots, \partial f/\partial x_n(P)) \cdot v = 0 \]
for all $f \in I(X)$.

Exercise 2. Do Exercise 4.7 of Chapter I of Hartshorne.

Exercise 3. Do Exercise 5.2 of Chapter I of Hartshorne.

Exercise 4. Let $Y$ be the $2 \times 3$ matrices of rank at most 1.

(a) Find the singular points of $Y$.
(b) For each nonsingular point $P$ of $Y$, give a pair of polynomials which suffices to define $Y$ locally near $P$. 