Exercise 1. Show that the graph $\Gamma$ of a morphism $X \to Y$ is a subprevariety of $X \times Y$, and the first projection induces an isomorphism of $\Gamma$ onto $X$. If $Y$ is a variety, show also that $\Gamma$ is closed in $X \times Y$.

Exercise 2. Let $Y$ be a nonempty algebraic set in projective space, and $X$ the affine cone over $Y$.

(a) Show that $X$ consists of the origin together with the preimage of $Y$ under the natural map $\mathbb{A}^{n+1}_k \setminus \{0\} \to \mathbb{P}^n_k$. In particular, there is a natural surjective map

$$\varphi : X \setminus \{0\} \to Y.$$ 

(b) Show that $\varphi$ is continuous.

(c) Show that $\varphi$ is open (i.e., the image of an open subset of $X \setminus \{0\}$ is an open subset of $Y$).

(d) Show that $X$ is irreducible if and only if $Y$ is irreducible.

Exercise 3. Let $Y$ be a projective variety, and $X$ the affine cone over $Y$.

(a) Show that $\dim \mathbb{P}^n_k = n$.

(b) Show that $\text{codim}_{\mathbb{A}^{n+1}_k} X = \text{codim}_{\mathbb{P}^n_k} Y$, and $\dim X = \dim Y + 1$.

(c) Conclude that if $\text{codim}_{\mathbb{P}^n_k} Y = 1$, then $Y = Z_h(F)$ for some homogeneous polynomial $F$.

Exercise 4. Let $X, Y \subseteq \mathbb{P}^n_k$ be projective varieties, and suppose that $\text{codim}_{\mathbb{P}^n_k} X + \text{codim}_{\mathbb{P}^n_k} Y \leq n$. Then show that $X \cap Y \neq \emptyset$, and every irreducible component $Z$ of $X \cap Y$ satisfies

$$\text{codim}_{\mathbb{P}^n_k} Z \leq \text{codim}_{\mathbb{P}^n_k} X + \text{codim}_{\mathbb{P}^n_k} Y.$$