Exercise 1. Let $K/F$ be any algebraic extension, with char $F = p$.
   (a) Prove that $L = \{ \alpha \in K : \alpha \text{ is separable over } F \}$ is a subfield of $K$ containing $F$.
   (b) Prove that no element of $K \setminus L$ is separable over $L$.
   (c) Prove that for every $\alpha \in K$, for some $n \geq 1$ we have $\alpha^{p^n} \in L$. (That is, $K$ is obtained from $F$ by taking $p$th-power roots)

The $L$ of the above exercise is called the separable part of $K/F$. Given a finite extension $K/F$, define the separable degree $[K : F]_s$ to be the degree over $F$ of the separable part of $K/F$.

Exercise 2. Show that $[K : F]_s$ is multiplicative in towers of extensions.

Exercise 3. Let $F$ be a finite field. Prove that every function $F \to F$ can be realized by a polynomial $f(x) \in F[x]$.

Exercise 4. Prove that the only automorphism of $\mathbb{R}$ is the identity. Hint: show that the condition $x > y$ can be expressed purely algebraically.

Exercise 5. Given a field $F$, determine the fixed field of the automorphism of $F(x)$ given by $x \mapsto x + 1$. 
