In these exercises, for \( p \) an odd prime, and \( j > 0 \), \( \left( \frac{j}{p} \right) \) denotes the Legendre symbol, defined to be 1 if \( j \) is a quadratic residue modulo \( p \) and \(-1\) otherwise.

**Exercise 1.** Show that if \( p \equiv 1 \pmod{4} \), that
\[
\prod_{0 < j < \frac{p}{2}} \left( \frac{\sin \frac{\pi j}{p}}{\left( \frac{j}{p} \right)} \right) > \prod_{0 < j < \frac{p}{2}} \left( \frac{\sin \frac{\pi j}{p}}{\left( \frac{p-j}{p} \right)} \right).
\]

Note that the number of quadratic residues and non-residues in \( (0, \frac{p}{2}) \) is the same. Since \( \sin \) is monotone increasing in \( [0, \frac{\pi}{2}] \), this exercise shows that quadratic residues cluster in the first half of the interval \( (0, \frac{p}{2}) \).

**Exercise 2.** Show that for \( p \equiv 3 \pmod{4} \), and \( p > 3 \), the class number formula for \( \mathbb{Q}(\sqrt{-p}) \) may be rewritten as
\[
h_K = \frac{1}{2 - \left( \frac{2}{p} \right)} \sum_{0 < j < \frac{p}{2}} \left( \frac{j}{p} \right),
\]
and conclude that there are more quadratic residues in the interval \( (0, \frac{p}{2}) \) than non-quadratic residues (of course, this conclusion holds also for \( p = 3 \)).

Hint: use the behavior of \( \left( \frac{j}{p} \right) \) under \( j \mapsto p - j \), and compare the sums obtained by separating \( j \) first by size, and then by parity.