We begin by introducing the sheaf of analytic functions on $X_{an}$; this plays a role in understanding smooth and étale morphisms similar to that of the analytic topology for proper and separated morphisms.

**Exercise 1.** Let $X$ be a scheme of finite type over Spec $\mathbb{C}$, with associated analytic topological space $X_{an}$.

(a) First suppose that $X$ is affine, and fix a closed imbedding $X \subseteq \mathbb{A}^n_{\mathbb{C}}$, so that $X_{an} \subseteq \mathbb{C}^n$ has the subset topology. Let $I$ be the ideal of polynomials defining $X$. For any open $U \subseteq X_{an}$, we define

$$O_{X}^{an}(U) = (\lim_{\to} O_{\mathbb{A}^n_{\mathbb{C}}}(V))/(I),$$

where $V$ ranges over open subsets (in the analytic topology) of $\mathbb{C}^n$ such that $V \cap X = U$, and $O_{\mathbb{A}^n_{\mathbb{C}}}$ denotes the sheaf of analytic functions on $\mathbb{A}^n_{\mathbb{C}}$ in the usual sense. That is, we take the $\mathbb{C}$-algebra of all analytic functions defined on some $V$, with two equivalent if they agree on some smaller $V$, and we mod out by the ideal generated by $I$.

Show that this gives a well-defined sheaf on $X_{an}$, which depends only on $X$ and not the imbedding into $\mathbb{A}^n_{\mathbb{C}}$. Conclude that for any $X$, we have a unique sheaf of $\mathbb{C}$-algebras $O_{X}^{an}$ defined by the condition that for any open affine $U$, we have that $O_{X}^{an}|_U$ agrees with the sheaf $O^{an}|_U$ constructed above.

(b) Show that $O_{X}^{an}$ gives $X_{an}$ the structure of a locally ringed space, with local rings having residue field $\mathbb{C}$, and that for any $x \in X_{an}$, the residue map $O_{X,x}^{an} \to \mathbb{C}$ corresponds to evaluation of functions at $x$.

(c) Show that a morphism $f : X \to Y$ of schemes of finite type over Spec $\mathbb{C}$ gives a morphism of locally ringed spaces, but not necessarily conversely. [This is a fancy way of saying that every polynomial is an analytic function, but not conversely]

**Exercise 2.** Do Hartshorne, Exercise 10.1 of Chapter III.

The following exercises form the beginnings of deformation theory.

**Exercise 3.** Do Hartshorne, Exercise 8.6 of Chapter II, but using the relationship between regularity and smoothness (write this out carefully, and discuss which hypotheses are and are not necessary).

**Exercise 4.** Do Hartshorne, Exercise 8.7 of Chapter II.