Exercise 1. (Chevalley’s theorem) Do Hartshorne, Exercise 3.19 of Chapter II.

Exercise 2. (Chow’s lemma) Do Hartshorne, Exercise 4.10 of Chapter II.

Exercise 3. Show that a topological space \( X \) is Hausdorff if and only if the diagonal map \( X \to X \times X \) is closed.

Exercise 4. (Extra credit) Show that a topological space \( X \) is quasi-compact if and only if for every topological space \( Z \), the projection map \( X \times Z \to Z \) is closed.

For the following exercises, let \( X \) be of finite type over \( \mathbb{C} \). You may use without proof (we will give a proof of this later) the following:

Theorem 5. Let \( U \) be a Zariski open subset of \( X \). The Zariski closure \( \bar{U} \subseteq X \) has preimage in \( X_{\text{an}} \) precisely equal to the closure (in the analytic topology) of the preimage of \( U \).

Exercise 6. Show that if \( X \) is disconnected in the Zariski topology, then \( X_{\text{an}} \) is disconnected in the analytic topology.

Exercise 7. Show that a subset \( U \) of \( X \) is Zariski closed if and only if it is constructible and its preimage in \( X_{\text{an}} \) is closed.

Hint: for the following two exercises, you will need to use Chevalley’s theorem and Chow’s lemma.

Exercise 8. Show that \( X \) is separated over \( \text{Spec} \, \mathbb{C} \) if and only if \( X_{\text{an}} \) is Hausdorff.

Exercise 9. Show that \( X \) is proper over \( \text{Spec} \, \mathbb{C} \) if and only if \( X_{\text{an}} \) is compact (i.e., quasi-compact and Hausdorff).

Exercise 10. Show that \( \text{Spec} \, \mathbb{C} \) is not universally closed. Hint: consider \( Z = \text{Spec} \, \mathbb{Z} \).