Allowing a slight abuse of terminology, we say that a scheme $X$ is **generically reduced** if there is a dense open subset $U$ which is reduced. The goal of this problem set is to prove the following classical statement, which ties our definition of degree to the classical one:

**Theorem 1.** Let $X \subseteq \mathbb{P}^n_k$ be a generically reduced projective scheme of dimension $m$ over an algebraically closed field $k$. Then a general linear subspace of $\mathbb{P}^n_k$ of dimension $n - m$ intersects $X$ in a finite set of reduced points.

Recall that “general” in this context means that there exists a non-empty open (necessarily dense) subset of the appropriate Grassmannian such that every linear space in this subset satisfies the desired condition.

**Exercise 2.** Show that if $X \subseteq \mathbb{P}^n_k$ has dimension $m$, then a general linear subspace of dimension $n - m - 1$ does not meet $X$. Also show that a general linear subspace of dimension $n - m$ meets $X$ in dimension 0.

*Hint:* you can use a classical dimension-counting argument via an “incidence correspondence”, considering the set of pairs $Z \subseteq \mathbb{P}^n_k \times \mathbb{G}(n - m - 1, n)$ of the form $(P, L)$ where $P$ is a point of $X$, and $L$ is a linear subspace of dimension $n - m - 1$ containing $P$.

**Exercise 3.** Use Corollary II.8.16 and Theorem II.8.18 of Hartshorne to prove the desired result; note that the argument for the latter result works more generally for quasiprojective varieties, if we weaken the statement slightly to assert that a general hyperplane has non-singular intersection with the given variety (i.e., instead of requiring that the locus of such hyperplanes is an open dense set, we only require that it contains an open dense subset).