DEFORMATION THEORY IN ALGEBRAIC GEOMETRY

Organizers: Max Lieblich (Princeton), Martin Olsson (Berkeley), Brian Osserman (Berkeley), Ravi Vakil (Stanford).

Advisory Committee: Brendan Hassett (Rice), Sándor Kovács (Washington).

1. Scientific purpose

Modern algebraic geometry offers a number of very powerful tools for solving a range of problems. Unfortunately, these tools can also seem overwhelmingly technical to a young student. This workshop aims to bridge the gap between the young graduate student starting out in algebraic geometry and the researcher in the subject.

The topic for the workshop is deformation theory, which touches upon many of the central themes of the subject: the notion of moduli and moduli spaces, representability of functors, algebraic spaces and stacks, singularity theory, obstruction theories and Gromov-Witten theory, and cohomology. Despite, or perhaps because of this, there is a shortage of accessible references which treat the topic in a unified manner, and as a result, most people learn it only by word of mouth. This workshop is therefore an opportunity to disseminate some degree of expertise in deformation theory to a much wider audience than is typically possible.

We also hope that the participants of the workshop are ultimately better prepared to fully participate in and benefit from the MSRI program in algebraic geometry in spring 2009.

2. Achievements

The workshop drew roughly 60 participants, and also included 8 teaching assistants; these were advanced or finishing graduate students with a very strong technical background in algebraic geometry. A number of additional local students sat in on many of the lectures. All participants attended lectures, gave short lectures to their work groups on background material from algebraic geometry, and worked with their groups on intensive daily problem sets. Fridays were devoted to guest lectures by prominent lecturers who illustrated how the technical machinery covered in the workshop has been applied to research problems in algebraic geometry.

Written lecture notes typed in real time by several volunteers are now posted to the workshop web site, along with the streaming video of the lectures produced by MSRI. In addition, the organizers are currently in the process of polishing and expanding the existing lecture notes into a book intended as an introduction to the field.

3. Description of workshop

Structure. The mathematical content of the workshop was anchored by three lecture series given by Lieblich, Olsson, and Osserman. These lectures series occupied half the day Monday through Thursday each week, and their topics were as follows:

2. Deformations (a): tangent and obstruction spaces (Olsson).
3. Deformations (b): representability and Schlessinger’s criterion (Osserman).

On Fridays we instead offered guest lectures by Jason Starr, Ravi Vakil, Paul Hacking and Brian Conrad, who gave 90-minute lectures each describing how deformation theory has been
applied to a different research problem. The topics of these lectures were rational curves on varieties, singularities of moduli spaces, topology of complex varieties, and Galois representations, respectively.

Outside of lecture, the participants were assigned to workgroups with 8 people each, with each group led by one of the TAs. These groups worked on the daily problem sets together. These sets consisted of exercises tightly integrated into the daily lectures, ranging from working out examples to filling in theoretical arguments to developing ideas beyond those treated in lecture. During this time, the students also had the opportunity to discuss the lectures with each other and with their TA. During the first week, students also short presentations on background material to their workgroups.

**Student presentations.** Every student gave one half-hour presentation to their workgroup on an assigned background topic in algebraic geometry which is important to learning deformation theory. The intent of this was two-fold: first, that the students in the workgroup would learn something about the topic from the lecture, and second, that the lecturer on each topic would then we someone (in addition to the group TA) that participants could consult on this topic later as the need arose. The topics for the background lectures were the following:

1. Flatness, and the local criterion for flatness.
2. Smoothness, and the formal criterion for smoothness.
3. Etaleness, and the formal criterion for etaleness.
4. Categories and functors; abelian categories.
5. Derived functors.
7. Cohomology on curves: Riemann-Roch and examples.
8. Grothendieck’s existence theory for coherent sheaves.

**Teaching assistants.** The role of the teaching assistants was essentially supervisory: they oversaw the background lectures and group work, and used their expertise to answer questions on the lectures and to assist in solving the problem sets. Although many of the TAs had some prior experience with deformation theory, they nonetheless acted also as participants, attending the lectures and learning the parts of the theory with which they were not yet familiar.

4. **Publications and presentations**

As mentioned above, online lecture notes and streaming videos are available on the workshop web page.

In addition, the organizers are in the process of writing an introductory book based on the lecture notes. Vakil is writing an introductory chapter, Lieblich is writing on descent theory and stacks, Olsson is writing on tangent and obstruction spaces and the cotangent complex, and Osserman is writing on Schlessinger’s criterion. In addition, Fred van der Wyck, one of the workshop participants, has prepared an appendix containing calculations of several examples in deformation theory. The organizers also hope to include in some form the material covered by the guest lectures, as well as a presentation of Artin’s criterion for representability of algebraic stacks, although the level of detail for the latter remains to be determined.