MAT 00C, Fall 2015
Midterm 2

Instructions:

1. Do not open this exam until you are told to do so.

2. Show all work for full credit. A correct answer with little or no work may not receive full credit.

3. No notes, books, electronic devices, or calculators may be used as resources for this exam.

4. It is a violation of the university honor code to, in any way, assist another person in the completion of this exam, copy answers from another student’s exam, or have another student take your exam for you. Any violations against the university honor code may be reported to Student Judicial Affairs.

5. Stop immediately when time is called and close your exam. Failure to do so may lead to a points deduction on your exam score.

Good luck!

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>/7</td>
</tr>
<tr>
<td>2</td>
<td>/7</td>
</tr>
<tr>
<td>3</td>
<td>/5</td>
</tr>
<tr>
<td>4</td>
<td>/9</td>
</tr>
<tr>
<td>5</td>
<td>/9</td>
</tr>
<tr>
<td>6</td>
<td>/17</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>/54</strong></td>
</tr>
</tbody>
</table>
1. (7 points) Sketch the graph of \( y = -3 \cos(2\pi x - \pi) \). Include two full periods.

Amplitude = 3
Period = \( \frac{2\pi}{2\pi} = 1 \)
Quarter Period = \( \frac{1}{4} \)
Phase Shift = \( \frac{\pi}{2\pi} = \frac{1}{2} \)

<table>
<thead>
<tr>
<th>Start</th>
<th>QP</th>
<th>HP</th>
<th>TP</th>
<th>FP</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\frac{1}{2}, -3))</td>
<td>(3(\frac{3}{4}, 0))</td>
<td>(1, 3)</td>
<td>(5(\frac{3}{4}, 0))</td>
<td>(3(\frac{3}{2}, -3))</td>
</tr>
</tbody>
</table>

\[ y = -3 \cos(2\pi x - \pi) \]

\[ \frac{1}{2} - 1 = -\frac{1}{2} \]
2. (7 points) Sketch the graph of $y = 2\sin(3x) + 1$. Include two full periods.

Amplitude = 2
Period = $\frac{2\pi}{3}$
Quarter Period = $\frac{\pi}{6}$
Phase Shift = 0.

<table>
<thead>
<tr>
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<th>QP</th>
<th>HP</th>
<th>TP</th>
<th>FP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 1)</td>
<td>($\frac{\pi}{6}, 3$)</td>
<td>($\frac{\pi}{3}, 1$)</td>
<td>($\frac{\pi}{2}, -1$)</td>
<td>($\frac{2\pi}{3}, 1$)</td>
</tr>
</tbody>
</table>

$y = 2\sin(3x) + 1$. 

\[ \text{\begin{array}{c}
-2\frac{\pi}{3} \\
0 \\
\frac{\pi}{6} \\
\frac{\pi}{3} \\
\frac{\pi}{2} \\
2\frac{\pi}{3}
\end{array}} \]
3. (5 points) Sketch the graph of \( y = 4 \tan \left( \frac{\pi x}{2} \right) \). Include two full periods.

\[
\begin{align*}
\frac{\pi x}{2} &= -\frac{\pi}{2} \\
\frac{\pi x}{2} &= \frac{\pi}{2}
\end{align*}
\Rightarrow
\begin{align*}
x &= -1 \\
x &= 1
\end{align*}
\] vertical asymptotes.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -1 )</th>
<th>( -\frac{1}{2} )</th>
<th>( 0 )</th>
<th>( \frac{1}{2} )</th>
<th>( 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 4 \tan \left( \frac{\pi x}{2} \right) )</td>
<td>und</td>
<td>-4</td>
<td>0</td>
<td>4</td>
<td>und</td>
</tr>
</tbody>
</table>

\[
y = 4 \tan \left( \frac{\pi x}{2} \right).
\]
4. (9 points) Sketch the graph of \( y = \sec(3x) \). Include two full periods.

\[
\begin{align*}
y &= \cos(3x) \\
\text{Amplitude} &= 1 \\
\text{Period} &= \frac{2\pi}{3} \\
\text{Quarter Period} &= \frac{\pi}{6} \\
\text{Phase Shift} &= 0.
\end{align*}
\]

\[
\begin{array}{c|c|c|c|c}
\text{Start} & \text{QP} & \text{HP} & \text{TP} & \text{FP} \\
(0, 1) & (\pi/6, 0) & (\pi/3, -1) & (\pi/2, 0) & (2\pi/3, 1)
\end{array}
\]
5. (9 points) Sketch the graph of $y = -\csc(\pi x + \pi)$. Include two full periods.

$$y = -\sin(\pi x + \pi)$$
Amplitude = 1
Period $= \frac{2\pi}{\pi} = 2$
Quarter Period = $\frac{1}{4}$
Phase Shift = $-\frac{\pi}{\pi} = -1$. 

<table>
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<th>HP</th>
<th>TP</th>
<th>FP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-1, 0)$</td>
<td>$(-\frac{1}{2}, -1)$</td>
<td>$(0, 0)$</td>
<td>$(\frac{1}{2}, 1)$</td>
<td>$(1, 0)$</td>
</tr>
</tbody>
</table>
6. Evaluate each expression.

(a) (2 points) \( \sin(\arcsin 0.5) = \frac{1}{2} \)

(b) (3 points) \( \arctan(\tan(\frac{11\pi}{6})) = \arctan(\tan(-\pi/6)) = -\pi/6 \)

(c) (2 points) \( \cos(\arccos(-3)) \) does not exist since \( \arccos(-3) \) is undefined (-3 is not in the domain of \( \arccos \)).

(d) (4 points) \( \sin(\arccos(-\frac{1}{3})) = \sin(\theta) = (+) \frac{2\sqrt{2}}{3} \)
\[ \theta = \arccos(-\frac{1}{3}) \]
\[ \cos \theta = -\frac{1}{3} \]
\[ z^2 = y^2 + 1 \]
\[ z = y^2 \]
\[ 2\sqrt{2} = y \]
\[ \cos \theta < 0 \) and \( 0 < \theta < \pi \)
imply \( \theta \) lies in Quadrant II.

(e) (3 points) \( \csc(\arctan(-3)) = \csc(\theta) = (-) \frac{\sqrt{10}}{3} \)
\[ \theta = \arctan(-3) \]
\[ \tan \theta = -3 \]
\[ \c^2 = 10 \]
\[ c = \sqrt{10} \]
\[ \tan \theta < 0 \) and \( -\frac{\pi}{2} < \theta < \frac{\pi}{2} \)
imply \( \theta \) lies in Quadrant IV.

(f) (3 points) \( \sec(\arcsin(x^2 - 1)) = \sec(\theta) = \frac{1}{\sqrt{2x-x^2}} \)
\[ \theta = \arcsin(x^2 - 1) \]
\[ \sin \theta = x^2 - 1 \]
\[ a^2 = (x^2 - 1)^2 \]
\[ a^2 = 1 - (x^2 - 1)^2 \]
\[ a^2 = -x^4 + 2x^2 \]
\[ a = \sqrt{2x-x^2} \]