Some practice problems for double integrals
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1. Evaluate the double integral
\[ \int \int_R (\sqrt{x} - y^2) \, dx \, dy \]
where \( R \) is the region enclosed by the curve \( y = x^2 \) and the curve \( y = x^{1/4} \).

2. Evaluate the double integral
\[ \int \int_R \cos \left( \frac{\pi}{2} x^2 \right) \, dx \, dy \]
where \( R \) is the triangle enclosed by the line \( y = x \), the vertical line \( x = 1 \) and the \( x \)-axis.

3. Evaluate the following double integrals.
(a) \( \int_0^1 \int_{x^2}^x xy^2 \, dy \, dx \).
(b) \( \int_{\pi/2}^{\pi} \int_0^{x^2} \frac{1}{2} \cos \left( \frac{y}{x} \right) \, dy \, dx \).
(c) \( \int_0^1 \int_{4}^{4x} e^{-y^2} \, dy \, dx \).

4. Sketch the solid in 3D-space whose volume is given by the double integral
\[ \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (7 - x^2 - y^2) \, dy \, dx. \]